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A Discussion of Log Rules

Their Limitations and Suggestions for Correction

BY
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PREFACE

THE lumberman is beginning to realize the necessity for standardizing the methods employed in handling his industry. We recognize the problem of standardization as a broad one and feel that the following discussion of log rules is an appropriate contribution to the solution of a problem which influences both the commercial handling of lumber and the scientific study of forest products. There is an unquestionable need for a standard rule for the accurate determination of the volume of logs of various lengths and diameters, and the amount of manufactured lumber possible to produce from such logs. There are many log rules in use throughout the United States, some more accurate than others.

The following discussion has been prepared by Mr. H. E. McKenzie, Forest Engineer with this department, and was suggested by the result of a mill scale study (to be issued as a separate publication) in which the statute rule of California, the Spaulding Log Rule, was found to show a marked discrepancy between the log scale and the amount of lumber sawed out. This discrepancy led to the further investigation embracing all of the log rules in use in the United States, with the view of determining what rule, if any, is universally applicable or to devise such a rule.

G. M. HOMANS,
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INTRODUCTION

IT is customary among the lumbermen of this country, when buying or selling logs, to base their calculations upon the value of the lumber the logs will produce when sawed rather than upon the total volume. The by-products, such as slabs, sawdust, and loss by normal crook, which accompany the manufacture of lumber from logs of various sizes, are therefore ignored in the valuation, and tables have been compiled which aim to show the volume of lumber in units, known as board feet (1″x 12″x 12″), after the elimination of by-products has been made. Such tables are called "log rules."

It is the object of this publication to discuss many of the different log rules now in use, to show the principles upon which they are based, and wherein they are defective; to introduce a new log rule, based upon mathematical principles, and designed to be flexible to the varying conditions, both in milling operations and in the character of the timber to be sawed. Also, to show relations, where they exist, between any two rules or any number of rules, such that a transformation from one rule to another can be accomplished, and to reduce the various rules, wherever possible, to a definite form, in order that comparisons by formulæ may be easily made, and the allowance for slabs, sawdust, etc., by each rule readily ascertained.

CONSTRUCTION AND UNDERLYING PRINCIPLES OF LOG RULES.

LOG RULES IN GENERAL.

About forty-five log rules have been devised within the last seventy-five years for the measurement of sawed lumber from logs of different sizes, and the values shown by these different rules cover an enormous range. It is safe to say that 90 per cent of them are so constructed that at best they are of value only under the conditions of the locality where they were first employed, and there is no means whereby they can be intelligently corrected for other conditions. Such is the case with all log rules based upon diagrams showing the amount of lumber in logs after allowances have been made for slabs, saw-kerf, etc. Such is the case with all log rules obtained by correcting these rules or combining them for others. Also rules resulting from actual experience at saw-mills have the same objections. They bear the prints of local conditions and, due to the method whereby they came into existence, they can never be anything more than local, and can only be applied to milling conditions similar to those existing at the mills where they were first constructed.

The only logical way of constructing a log rule which will be flexible and which will adjust itself to universal conditions, is to so construct it that the underlying, fundamental principles are so segregated as to make them independent of one another, and to have them so worked together as to give the aggregate result of all factors, which will be in all cases proportional and equal to the volume of the manufactured product. There are several distinct principles underlying the measurement of lumber which logs of different sizes will produce, which cannot be overlooked in any rule that is destined to become a correct universal measure. Such a rule must embody the principle that the slabs which cover the material, or part of the log which is to become the finished product, should be allowed for by making the allowance proportional to the barked area of the log. The slabs are the covering, as it were, which necessarily has to be removed in order to get to the part of the log that produces lumber, and they should not be, and are not, cut any thicker from large logs than from small ones. The best material contained in the log usually lies nearest to the bark, and it is greatly to the advantage of the millman not to waste any of his best grades.

Several log rules in most common use today do not embody the above principle. The Spaulding Log Rule, which is the statute rule of California, does not adhere to it. The Scribner Rule, which is the official rule of the Forest Service, U. S. Department of Agriculture, and of several states, does not take it into consideration, and instead of having the volume of slabs proportional to the barked area of the logs, they have them proportional to the total volume, as will be shown further on.

It would not be any more absurd if one tried to figure the number of board feet necessary to side up a house by figuring the volume of the house instead of its lateral surface. A definite per cent cannot be given as indicating the relation of slabs to trees of different volume, any more than a definite per cent can be given as indicating the relation of all lateral surface to the volume of houses of different dimensions. The Spaulding, Scribner and all other log rules with a waste allowance for slabs varying directly as the volume of the log are mathematically incorrect, since there is no reason for cutting any thicker slabs from large logs than from small ones.

Another principle underlying the measurement of lumber contained in logs of different diameters and lengths is the relation of the allowance for sawdust to the size of the log. Since the waste allowance which should be allotted to slabs should be proportional to the barked area, it can be met by reducing the diameter of all sized logs a constant amount, and the remaining volume can then be considered as lumber plus sawdust. It is very evident that the sawdust allowance depends upon the dimensions of the lumber to be sawed and upon the width of the saw used. It is also evident that, for any specific width of saw-kerf and dimensions of lumber to be sawed, the allowance for sawdust should be a definite per cent of the total volume of all logs, not including slabs. A sawdust factor which fulfills these conditions is as follows:

$$\frac{k\,(w + t + k)}{(w + k)\,(t + k)}$$

Where k = width of saw, in inches.
w = average width of lumber to be manufactured, in inches.
t = average thickness, in inches.

This factor shows what fractional part of the log minus allowance for slabs should be allowed for sawdust.

$$\left[1 - \frac{k\,(w + t + k)}{(w + k)\,(t + k)}\right]$$

represents the fractional part of the log after slab allowance is made, which becomes lumber.

Log rules which ignore these principles can not be any more than local rules, applying to conditions existing at very few mills.

There are several other considerations to be taken into account in constructing a log rule, which are not of such vital importance as the two principles cited above. They are allowances for taper, shrinkage, normal crook and excessive taper in small logs. All of these factors depend largely upon the character of the timber, and should be adjusted accordingly for the different species, and for the same species growing under different conditions.

THE THREE RULES MOST COMMONLY USED.

The Spaulding Log Rule.

The Spaulding Log Rule is the statute rule of California, having been adopted by an act of the legislature in 1878. It is constructed from diagrams, and the following comments upon it were published by its author:

> "Each sized log has been scaled so as to make all that can be practically sawed out of it, if economically sawed. Each log to be measured at the top of small end, inside of the bark, and if not round, to be measured two ways—at right angles—and the average

taken for the diameter. Where there are any known defects, the amount to be deducted should be agreed upon by the buyer and the seller, and no fractions of an inch to be taken into the measurement.

"In the foregoing table I have varied the size of the slab in proportion to the size of the log, and have arranged it more particularly for large logs by taking them in sections of twelve feet and carrying the table up to 96″ in diameter. As there has never been any in use for scaling over 44″, it has been my purpose to furnish a table for the measuring of logs that can be implicitly relied upon for correctness by both the buyer and the seller; and to do so, I have spared no pains to render it perfect."

This rule has been very carefully prepared, and all values given are very consistent with the principles upon which it is constructed. These principles are clearly shown in the graphic analysis made of the rule in Fig. 1. They are as follows: (a) The sawdust allowance varies

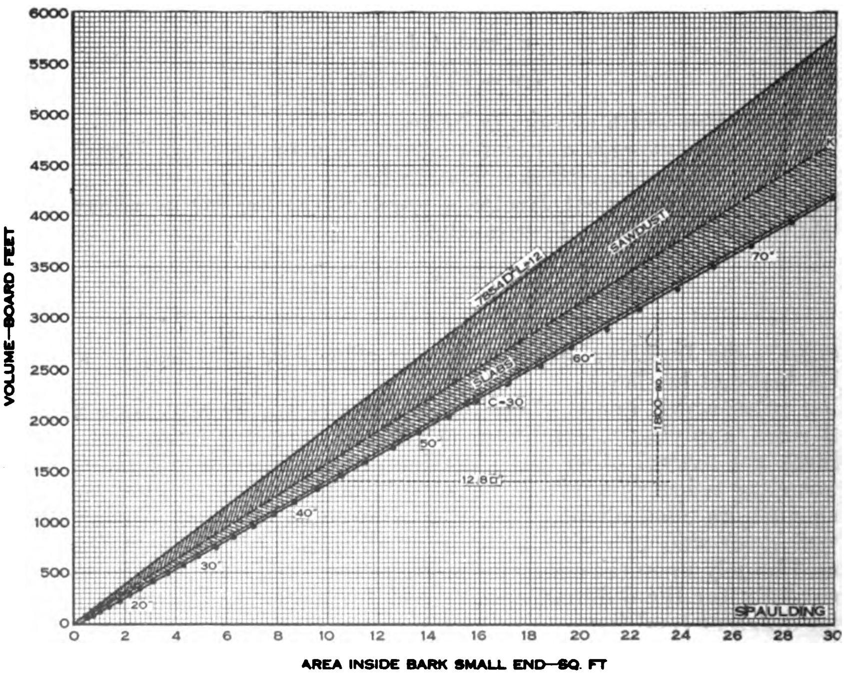

FIG. 1. A graphic analysis of the Spaulding Log Rule, based upon area in square feet inside bark at small end of logs. This diagram shows the following: (*a*) Top curve, total contents in board feet of logs of different diameters 16′ long with no allowance made for taper. (*b*) Curve "k," volume in board feet remaining after 18% of the total volume has been allowed for sawdust (this allowance is about right for ¼″ saw-kerf). (*c*) Curve passing through origin and drawn parallel to bottom curve. (*d*) Bottom curve located by plotting volumes in board feet for 16′ logs of even inches in diameter inside bark, as given by the Spaulding Log Rule. The formula indicated by this analysis is as follows: $(.048D^2 - 2)L$ = B. M. = volume in board feet.

directly with the volume. (b) Slab allowance varies directly as the volume plus a constant. (c) No allowance made for taper. (d) No allowance made for normal crook. (e) Total waste allowance remains constant, regardless of the width of saw-kerf.

The big disadvantage of such a rule lies in the fact that it is not flexible to conditions existing at mills in different localities where it might be used, or to the character of the timber sawed. It is unaffected by taper, normal crook, width of saw-kerf and excessive taper in small logs, and such corrections can not be properly made due to the diagram

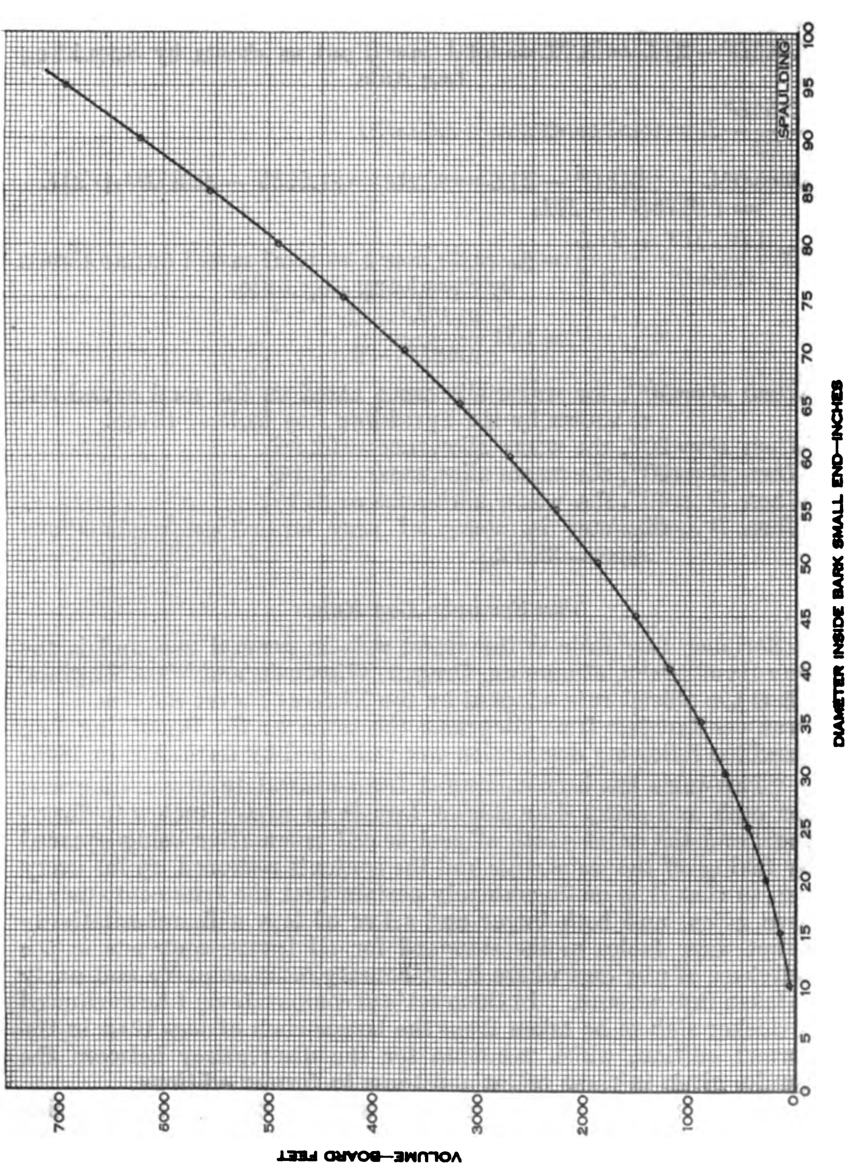

Fig. 2. The curve shown in this figure is a graphic representaton of the formula $(.048D^2 - 2)L = $ B. M., when $L = 16'$. The points on the curve are plotted independently from values given by the Spaulding Log Rule, the object being to show how closely the formula fits the rule.

method used in first constructing the rule. Fig. 1 indicates the following formula: $(.048D^2 - 2)L = $ B. M. = volume in board feet, which very closely fits this rule as shown in Fig. 2.

Small logs will invariably over-run this scale, due to the constant "2" shown by the formula. Intermediate logs will hold up the scale, fall below, or go above, largely depending upon the width of saw-kerf and the average dimensions of the lumber sawed. Large logs will generally run higher than the intermediate sizes, due to the fact that the slab allowance varies directly with the volume plus a constant. The following deduction shows the total waste allowance of the Spaulding Log Rule expressed in per cent of the rule:

$(.048D^2 - 2)L = \text{B. M.} = $ total sawed out as shown by Spaulding Log Rule.

$$\frac{.7854D^2}{12}L = \text{total contents} = .0655D^2L$$

$$.0655D^2L - (.048D^2 - 2)L = \text{waste} = [(.0655 - .048)D^2 + 2]L = (.0175D^2 + 2)L.$$

$$100\,\frac{(.0175D^2 + 2)\,L}{(.048D^2 - 2)\,L} = \%\ \text{waste based on total sawed out as shown by Spaulding Log Rule.}$$

$$= 100\,\frac{.0175D^2 + 2}{.048D^2 - 2}.$$

When $D = 10''$, the waste allowance based on the total sawed out as shown by the Spaulding Log Rule $= 134\%$.
When $D = 20''$, the waste allowance $= 52.2\%$.
When $D = 30''$, the waste allowance $= 43.1\%$.
When $D = 40''$, the waste allowance $= 40.1\%$.
When $D =$ diameter in inches of very large logs, waste allowance $= 36.5\%$.

The Scribner Log Rule.

The Scribner Log Rule is the oldest rule in general use, and is the statute rule of Idaho, Minnesota, Oregon, Wisconsin and West Virginia. Also, it is the official rule adopted by the Federal Forest Service.

It was constructed from diagrams the same as the Spaulding Log Rule, and the following description was published by its author in 1846:

"This table has been computed from accurately drawn diagrams for each and every diameter of logs from twelve inches to forty-four, and the exact width of each board taken after being squared by taking off the wane edge and the contents reckoned up for every log, so that it is mathematically certain that the true contents are here given, and both buyer and seller of logs will unhesitatingly adopt these tables as the standard for all future contracts in the purchase of saw logs where strict honesty between party and party is taken into account. In these revised computations I have allowed a thicker slab to be taken from the larger class of logs than in the former edition, which accounts for the discrepancy between the results given in these tables and those in former editions.

"The diameter is supposed to be taken at the small end, inside the bark, and in sections of 15′, and the fractions of an inch not taken into the measurement. This mode of measurement, which is customary, gives the buyer the advantage of the swell of the log, the gain by sawing into scantling, or large timber, and the fractional part of an inch in the diameter. Still it must be remembered that logs are never straight and that oftentimes there are concealed defects which must be taken as an offset for the gain above mentioned. It has been my desire to furnish those who deal

> in lumber of any kind with a set of tables that can implicitly be relied upon for correctness by both buyer and seller, and to do so I have spared no pains nor expense to render them perfect; and it is to be hoped that hereafter these will be preferred to the palpably erroneous tables which have hitherto been in use. If there is any truth in mathematics or dependence to be placed in the estimates given in diagrams, there cannot remain a particle of doubt of the accuracy of the results here given.''

This log rule gives practically the same results as does the Spaulding. It is not as carefully prepared, however, since the values given are not as consistent with the underlying principles of the rule. A graphic

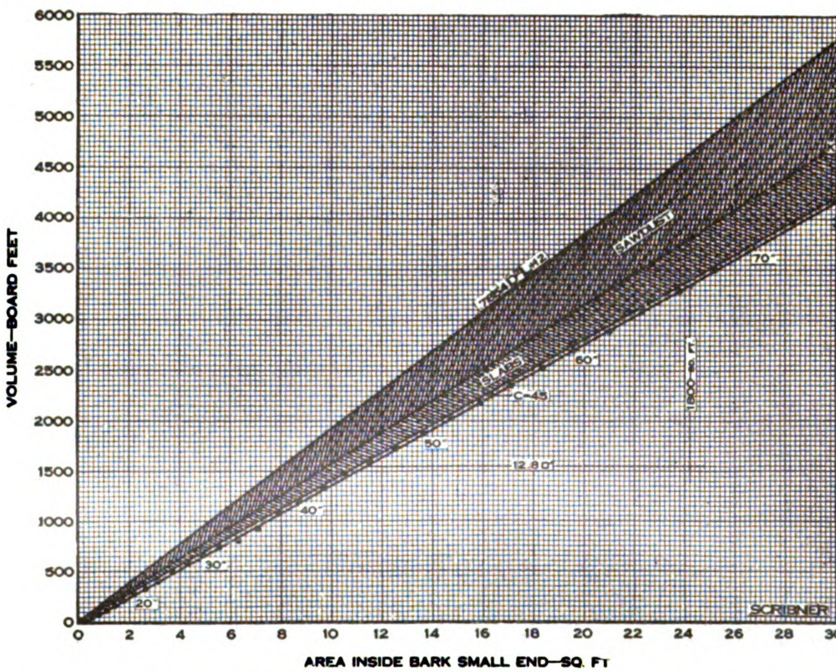

FIG. 3. A graphic analysis of the Scribner Log Rule, based upon area in square feet inside bark at small end of logs. This diagram shows the following: (*a*) Top curve, total contents in board feet of logs of different diameters 16′ long, with no allowance made for taper. (*b*) Curve "k," volume in board feet remaining after 18% of the total volume has been allowed for sawdust (this allowance is about right for ¼″ saw-kerf). (*c*) Curve passing through origin and drawn parallel to bottom curve. (*d*) Bottom curve located by plotting volume in board feet for 16′ logs of even inches in diameter inside bark as given by the Scribner Log Rule. The formula indicated by this analysis is as follows: $(.048D^2 - 3)L = \text{B. M.} =$ volume in board feet. This formula is almost identical with the one obtained for the Spaulding Log Rule. It does not apply, however, to diameters below 14″ or above 75″. No formula can be written for the Scribner Log Rule that will fit all values given, due to the inconsistency of the individual values of the rule.

analysis of it is given in Fig. 3, which shows the fundamental principles upon which it is based, and which are the same as for the Spaulding rule. The formula indicated by the analysis shown in Fig. 3 is $(.048D^2 - 3)L = \text{B. M.} =$ volume in board feet, which is practically the same as for the Spaulding Log Rule, the only difference being in the constant "3". Fig. 4 shows how closely this formula fits the rule.

Small logs will invariably overrun this scale, and to a slightly greater extent than for the Spaulding Log Rule, since the constant shown by the formula is "3" instead of "2". Intermediate logs will hold up the

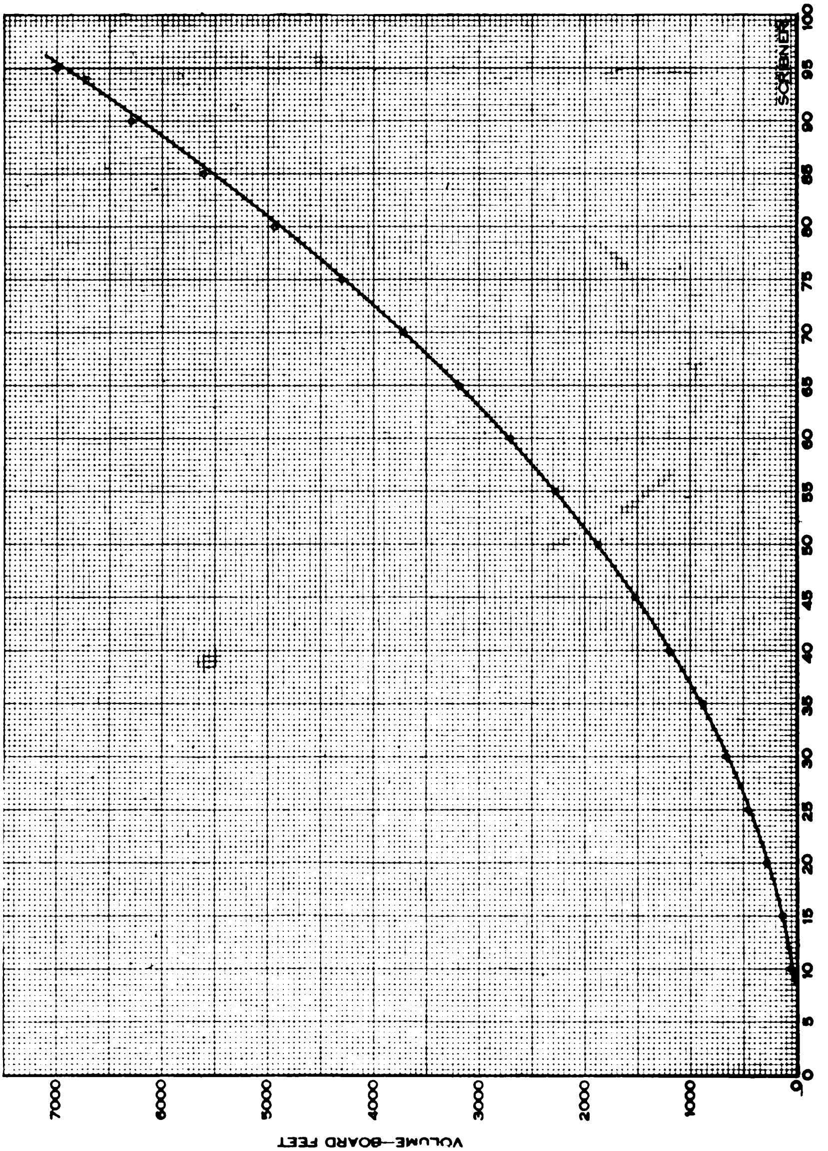

FIG. 4. The curve shown in this figure is a graphic representation of the formula $(.048D^2 - 3)L = $ B. M., when $L = 16'$. The points on the curve are plotted independently from values given by the Scribner Log Rule, the object being to show how closely the formula fits the rule. (It does not apply to diameters below 14" or above 75".)

scale, fall below or go above, largely depending upon the width of the saw-kerf and the average dimensions of the lumber sawed. Large logs will run higher than the intermediate sizes, due to the fact that the slab allowance is directly proportional to the volume plus a constant. The following deduction shows the total waste allowance of the Scribner rule expressed in per cent of total sawed out, as shown by the rule:

$(.048D^2 - 3)L = $ B. M. = Total sawed out as shown by the Scribner Log Rule.

$\frac{.7854D^2}{12}L = \text{total contents} = .0655D^2L$

$.0655D^2L - (.048D^2 - 3)L = \text{waste} = [(.0655 - .048)D^2 + 3]L$
$= (.0175D^2 + 3)L$

$100\frac{(.0175D^2 + 3)L}{(.018D^2 - 3)L} = $ % waste based on total sawed out as shown by Scribner Log Rule.

$= 100\frac{.0175D^2 + 3}{.048D^2 - 3}.$

When $D = 10''$, the waste allowance based on the total sawed out as shown by the Scribner Log Rule = (Formula does not apply below 14″).
When $D = 20''$, the waste allowance = 61.8%.
When $D = 30''$, the waste allowance = 46.7%.
When $D = 40''$, the waste allowance = 42.0%.
When D = diameter in inches for very large logs, waste allowance = 36.5%.

The Doyle Log Rule.

The Doyle Log Rule is used throughout the entire country and is the statute rule of Florida, Louisiana and Arkansas. It is constructed from the formula $\left(\frac{D-4}{4}\right)^2 L = $ B. M., which is stated as follows: Deduct 4″ from the diameter of the log as an allowance for slabs; square one quarter of the remainder and multiply the result by the length of the log in feet. No mention is made in this rule of a sawdust allowance. If four inches from the diameter of the small end is the slab allowance, the sawdust allowance must be the difference between the solid contents in board feet remaining after the slab allowance has been made and the contents shown by the rule. The determination of sawdust allowance follows:

$\left(\frac{D-4}{4}\right)^2 L = B.M. = $ volume in board feet, as shown by the Doyle rule, of log D inches in diameter at small end inside bark and L feet long.

$\frac{.7854\,(D-4)^2}{12}L = $ volume in board feet of log D inches in diameter inside bark at small end L feet long with waste allowance for slabs but none for sawdust.

$\frac{.7854\,(D-4)^2}{12}L - \left(\frac{D-4}{4}\right)^2 L = $ sawdust allowance for log D inches in diameter and L feet long.

$\dfrac{\frac{.7854\,(D-4)^2}{12}L - \left(\frac{D-4}{4}\right)^2 L}{\frac{.7854\,(D-4)^2}{12}L} \times 100 = $ sawdust allowance for log D inches in diameter and L feet long expressed in per cent of volume in board feet left after slab allowance has been made

$= \frac{.295}{.0655} = 4.5\%$

Therefore, the sawdust allowance for the Doyle Log Rule = 4.5% of the total volume left after 4″ has been deducted from the diameter as an allowance for slabs. This sawdust allowance is correct in principle, since it is a definite per cent of the total volume after slabs have been accounted for. It is, however, entirely too small. The thinnest modern band saws take away at least 10% of the volume of the lumber sawed unless the product be large timbers, and the allowance of 4.5% is not one-half as large as it should be for even one of these saws. The

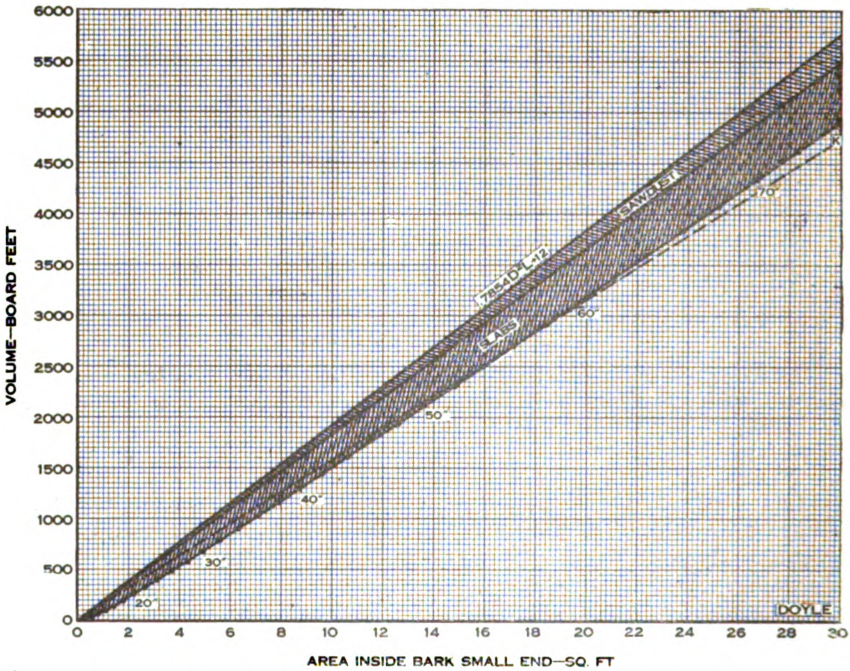

FIG. 5. A graphic analysis of the Doyle Log Rule, based upon area in square feet inside bark at small end of logs. This diagram shows the following: (*a*) Top curve, total contents in board feet of logs of different diameters 16′ long with no allowance made for taper. (*b*) Next lower curve, volume in board feet remaining after an allowance of 4.5% has been made for sawdust. (4.5% of the total volume of logs, after slab allowance has been made, is the only portion of the waste allowance of the Doyle Log Rule that varies directly as the volume. Therefore, it is the only part of the formula that varies directly as the amount of sawdust.) (c) Curve "k," values for volume in board feet after an allowance of 18% for sawdust has been made. This curve intersects the log rule at about 56″, showing, that, at this point and above, the waste allowance which should cover slabs and sawdust is not sufficient to even cover the sawdust. The Doyle Log Rule, however, is correct in principle, but its values are ver" poorly chosen.

principle upon which the Doyle Log Rule is based is correct, however, since the slab allowance is proportional to the barked area and the sawdust allowance is proportional to the total volume left after the allowance for slabs has been made. But the allowance for slabs is absurdly large and that for sawdust is absurdly low. In short, the principle of the rule is correct, but the values are very poorly chosen. Fig. 5 shows a graphic analysis of the rule.

A log rule was used long before the Doyle rule came into existence, which gave the same results, and was stated as follows: Deduct 4″ from the diameter for slabs, then, squaring the remainder, subtract one-fourth for saw-kerf and the balance will be the contents of the log 12′ long, from which the others may be obtained by proportion. It would appear from this that a generous allowance for sawdust had been made, but as a matter of fact the apparent sawdust allowance is a part of the allowance already made for slabs. This is clearly illustrated in figures 6 and 7, when the above rule is applied. (Deduct 4″ from the diameter for slabs and in Figures 6 and 7 we have $D - 4 = AB$. Then, squaring the remainder $(D - 4)$, we have $(D - 4)^2 = ABCD$. Subtract $\frac{1}{4}$

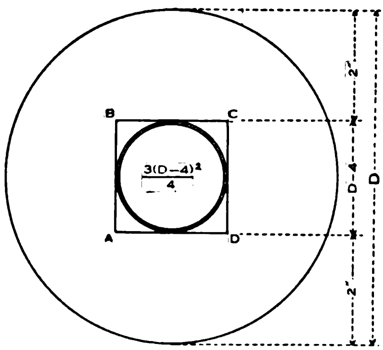

FIG. 6. The Doyle Log Rule as applied to a 6″ log.

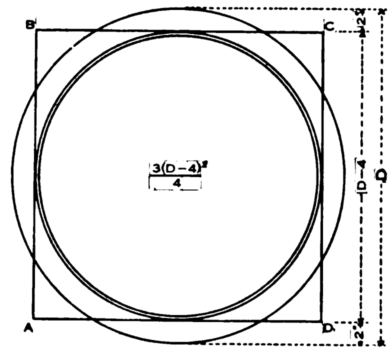

FIG. 7. The Doyle Log Rule as applied to a 30″ log.

for saw-kerf, giving $\frac{3}{4}(D - 4)^2$, which is the inside circle. The inscribed circle outside of this is equal to $.7854(D - 4)^2$. It is apparent from this that $.7854(D - 4)^2 - \frac{3}{4}(D - 4)^2$ is the only true portion of the diagram which could represent sawdust.) This rule amounts to the same thing as the Doyle Log Rule, but in statement is misleading and ambiguous.

The sawdust allowance as shown by Figures 6 and 7 in per cent of total contents after slab allowance has been made is as follows:

$$\frac{.7854\,(D - 4)^2 - \frac{3}{4}\,(D - 4)^2}{.7854\,(D - 4)^2} \times 100 =$$

$$\frac{.0354\,(D - 4)^2}{.7854\,(D - 4)^2} \times 100 = \frac{3.54}{.7854} = 4.5\%$$

which is the same as shown by the Doyle Log Rule formula.

The following deduction will show the total waste allowance of the Doyle Log Rule for logs of different sizes expressed in per cent of total sawed out, as indicated by the rule:

$\left(\frac{D-4}{4}\right)^2 L = B.M. =$ volume in board feet of log D inches in diameter at small end inside bark and L feet long.

$\frac{.7854\,D^2}{12} L =$ total volume in board feet contained in log D inches in diameter and L feet long. (No allowance for taper.)

$\frac{.7854\,D^2}{12} L - \left(\frac{D-4}{4}\right)^2 L =$ total waste allowance.

$$\frac{\frac{.7854\,D^2}{12} L - \left(\frac{D-4}{4}\right)^2 L}{\left(\frac{D-4}{4}\right)^2 L} \times 100 = \frac{.003\,D^2 + .5\,D - 1}{.0625\,D^2 - .5\,D + 1} \times 100.$$

= total waste allowance for log D inches in diameter and L feet long expressed in per cent of used volume.

When $D = 10''$, the waste allowance based on the total sawed out as shown by the Doyle Log Rule $= 191\%$.

When $D = 20''$, the waste allowance $= 63.8\%$.

When $D = 30''$, the waste allowance $= 39.5\%$.

When $D = 40''$, the waste allowance $= 29.4\%$.

When $D = 50''$, the waste allowance $= 23.8\%$.

This waste allowance is obviously too high for small logs and too low for large ones. This is due to the fact that the slab allowance is too generous and the sawdust allowance too small. Small logs will invariably over-run the scale; intermediate logs will usually scale about right, since the large slab allowance makes up the shortage for sawdust; large logs will invariably under-run the scale, because the combined slab and sawdust allowance is too small for waste, though the actual slab allowance is too large for slabs alone.

The McKenzie Log Rule.

The McKenzie Log Rule is based on mathematical principles and is designed to cover all conditions encountered in the manufacture of lumber from logs of various diameters and lengths. All factors influencing the total volume sawed out have been taken into consideration and treated separately, thus making the rule flexible to the varying conditions, both in milling operations and in the character of the timber.

The following factors which affect the mill output from logs of different sizes have been included:

(*a*) Slabs.
(*b*) Normal crook.
(*c*) Saw-kerf.
(*d*) Average dimensions of lumber sawed.
(*e*) Taper.
(*f*) Excessive taper in small logs.

The mathematical principles underlying the rule are as follows:

(*a*) The slab allowance is a function of the barked area and varies directly with it.

(*b*) Normal crook is also a function of the barked area, and varies directly with it the same as slabs.

(*c*) The sawdust allowance is a function of saw-kerf and average dimensions sawed at mill, and for any given saw-kerf and average dimensions the sawdust allowance should vary directly as the volume minus the slabs.

(*d*) Taper allowance equal to e'' in f'. (f not to exceed $16'$.)

(*e*) Excessive taper in small logs offset by a constant.

Let $D =$ diameter in inches inside bark at small end.
Let $L =$ length of log in feet.
Let $k =$ width of saw-kerf, in inches.
Let $w =$ average width of lumber sawed, in inches.
Let $t =$ average thickness of lumber sawed, in inches.
Let $C =$ constant.
Let $a =$ constant.

then $(D - a) =$ diameter of log after an allowance for slabs and normal crook has been made. (Since slabs and normal crook both vary the same, they can be accounted for by the same constant, a.)

$\frac{\pi (D - a)^2}{4} =$ area in square inches of small end of log after the slab and normal crook allowance has been made.

$\frac{\pi (D - a)^2 L}{4} =$ volume in units of $1'' \times 1'' \times 12''$ contained in log L feet long and D inches in diameter after the slab and normal crook allowance has been made. (Taper allowance to be made later.)

$\frac{\pi (D - a)^2 L}{4 \times 12} =$ volume in units of $1'' \times 12'' \times 12''$ or board feet in log L feet long and D inches in diameter after slab and normal crook allowance has been made.

No allowance has, as yet, been made for sawdust. This allowance depends upon the width of saw-kerf and the average dimensions of lumber to be sawed. The saw-kerf from one side and edge of an average board bears the same ratio to that board as the total sawdust from all boards does to the total volume after slab allowance has been made. This is true of all volume becoming sawdust, excepting saw-kerf amounting to $2k(D - a)$, which should be considered as part of the slabs since it varies directly as the barked area, and is the sawdust formed in cutting the slabs.

$$k\,(w + t + k)\,\frac{L}{12} =$$ volume of wood forming sawdust from each average board.

$$(w + k)\,(t + k)\,\frac{L}{12} =$$ volume of sawdust plus volume of average board.

$$\frac{k\,(w + t + k)\,\frac{L}{12}}{(w + k)\,(t + k)\,\frac{L}{12}} = \frac{k\,(w + t + k)}{(w + k)\,(t + k)} =$$ fractional part of wood, necessary to make average board, becoming sawdust.

This ratio of sawdust to average board plus sawdust holds for volume of logs minus allowance for slabs.

$$\left[1 - \frac{k\,(w + t + k)}{(w + k)\,(t + k)}\right] =$$ fractional part of log, after slab allowance is made, which becomes lumber.

Therefore, $\left[1 - \frac{k\,(w + t + k)}{(w + k)\,(t + k)}\right] \pi \frac{(D - a)^2}{48}.\ L =$ volume in board feet of lumber of average dimensions from log D inches in diameter at small end inside the bark and L feet long, when saw-kerf is k inches wide.

A constant $C =$ to a few board feet, when added to this formula has a compensating effect for the excessive taper in small logs. Since most small logs sawed are the top logs from medium or large sized trees, they have an excessive taper which can not be accounted for by a uniform taper allowance applied to the whole tree. Therefore, this constant, which in all cases will be very small (not exceeding 10 board feet) is applied and its effect on large logs is negligible, but on small ones it will play an important part in eliminating an accumulative error in total sawed out at the mill.

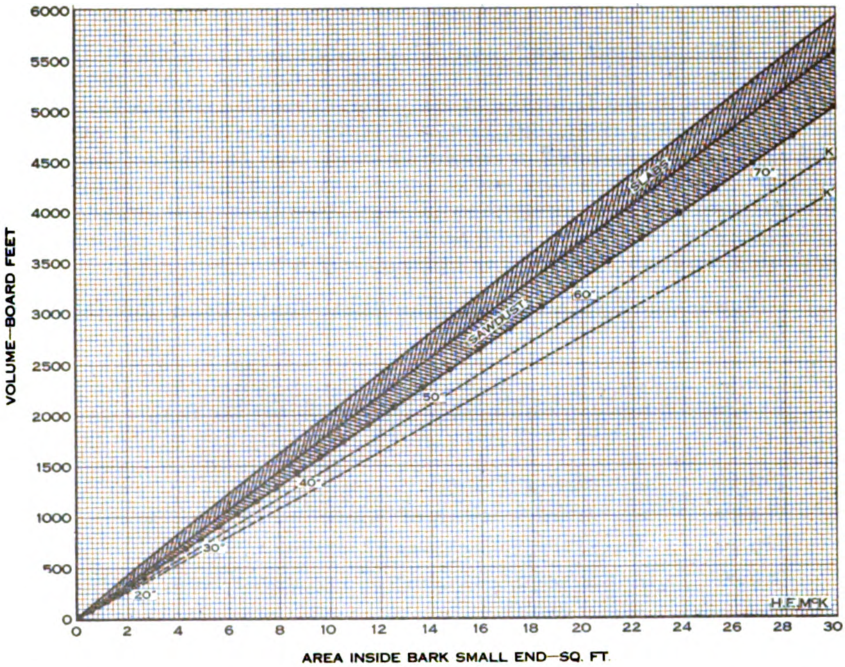

FIG. 8. A graphic analysis of the McKenzie Log Rule, based upon area in square feet inside bark at small end of logs. This diagram shows the following: (*a*) Top curve, total contents in board feet of logs of different diameters 16′ long with taper allowance of 1″ in 8′. (*b*) Next lower curve, volume in board feet remaining after an allowance for slabs has been made. (*c*) The log rule curve for ⅛″ saw-kerf, showing volume in board feet after an allowance for slabs and sawdust has been made. (The allowance for slabs in this rule varies directly as the "barked" area, and that for sawdust directly as the volume minus slab allowance.) (*d*) Curve "k," position that the log rule curve takes when the saw-kerf is ¼″ instead of ⅛″. (*e*) Curve "k′" shows position of the log rule curve for a ⅜″ saw-kerf. The formula for this rule is as follows:

$$\left[1-\frac{k(w+t+k)}{(w+k)(t+k)}\right]\frac{\pi(D-a)^2}{48}L+C=\text{B.M.}$$

k = width of saw-kerf, in inches..
w = average width of lumber, sawed, in inches.
t = average thickness of lumber sawed, in inches.
π = 3.1416.
D = average diameter inside bark, small end, in inches.
a = constant.
L = length of log, in feet.
C = constant included to compensate for excessive taper in small logs.

The formula is:

(not making any allowance for shrinkage and surfacing; the complete formula with this allowance made is shown on page 52.):

$$\left[1-\frac{k(w+t+k)}{(w+k)\ (t+k)}\right]\frac{\pi(D-a)^2}{48}L+C=\text{B.M.}$$

with a taper allowance of e'' in f' to be applied when compiling a table. The section used should not be taken over 16′ long: 8′ is better.

Its Application.

The above formula when applied to conditions existing at the Red River Lumber Company's mill in Lassen County, California, gave results shown in the following table. The value of a determined at this mill is extremely small, due to the fact that slabs were cut very thin and edgings were graded as moulding stock, also to the fact that short lengths were cut from logs where taper was great enough to permit it. The formula was first applied to 16′ logs, thus getting the taper in 16′ included with the slabs. Volumes in board feet of logs of other lengths were then figured with a taper allowance of 1″ in 8′.

TABLE 1. The McKenzie Log Rule, based upon the following formula:

$$\left[1 - \frac{k\,(w + t + k}{(w + k)\,(t + k)}\right] \frac{\pi\,(D - a)^2}{48} L + C = \text{B.M.}$$

Where k = saw-kerf = $\frac{1}{8}''$.
Where k = average width of lumber = 12″.
Where t = average thickness of lumber = $\frac{5}{4}''$.
Where D = average diameter of log inside bark, small end, in inches.
Where a = 1″.
Where L = length of log in feet.
Where C = 2 = constant allowed for excessive taper occurring in small logs.
Where B. M. = volume in board feet.
Where π = 3.1416.

With these values substituted, the formula becomes $.942(D - 1)^2 + 2 = \text{B. M.}$ for 16′ logs.

Table based upon 16′ logs. Taper allowance of 1″ in 8′ made for other lengths.

TABLE I.

Length in feet	4	5	6	7	8	9	10	11	12	13	14	15	16	Length in feet
	DIAMETER IN INCHES — BOARD FEET													
8	4	7	10	15	21	27	35	43	53	63	74	87	99	8
9	5	8	12	18	24	31	40	50	61	72	85	99	113	9
10	6	9	14	20	28	36	46	56	69	81	96	112	127	10
11	7	11	16	23	31	40	51	63	76	91	106	124	141	11
12	7	12	18	25	34	44	56	69	84	100	117	136	155	12
13	8	13	19	28	38	49	62	76	92	109	128	149	170	13
14	9	14	21	31	41	53	67	83	100	118	139	161	184	14
15	10	16	23	33	44	57	72	89	108	127	150	173	198	15
16	10	17	25	36	48	62	78	96	116	137	161	186	214	16
17	11	19	28	39	52	67	84	104	125	148	173	200	230	17
18	12	21	30	43	57	73	91	112	134	159	185	214	246	18
19	14	23	33	46	61	78	98	120	144	169	198	228	262	19
20	15	25	35	50	65	83	104	127	153	181	210	242	278	20
21	16	26	38	53	70	89	111	135	162	191	223	257	294	21
22	18	28	41	56	74	94	118	143	171	202	235	271	310	22
23	19	30	44	60	79	100	124	151	180	213	247	285	327	23
24	21	32	46	63	83	105	131	159	190	224	260	300	343	24
25	23	35	49	67	88	112	139	168	201	236	274	316	361	25
26	25	37	53	72	94	118	147	178	212	249	288	332	379	26
27	27	40	56	76	99	125	155	187	223	261	302	348	397	27
28	29	42	59	80	104	131	162	196	233	274	316	364	415	28
29	30	45	63	85	110	138	170	205	244	286	331	380	433	29
30	32	48	66	89	115	145	178	214	255	298	345	396	451	30
31	34	50	70	94	121	151	186	224	266	311	359	413	470	31
32	36	53	73	98	126	158	194	233	277	323	374	429	487	32

TABLE I—Continued.

Length in feet	17	18	19	20	21	22	23	24	25	26	27	28	29	Length in feet
	BOARD FEET													
8	114	129	114	162	180	199	219	240	261	283	307	332	358	8
9	130	147	164	184	205	226	249	273	296	321	348	377	406	9
10	146	165	185	207	230	254	279	305	332	360	390	421	454	10
11	162	183	205	230	255	281	309	338	367	398	431	466	501	11
12	178	201	225	252	280	309	339	370	403	436	473	511	550	12
13	195	219	245	275	304	336	369	403	438	475	515	556	598	13
14	211	237	266	297	329	363	399	436	473	513	556	600	645	14
15	227	255	286	319	354	391	429	469	509	552	597	645	693	15
16	243	273	307	342	378	417	457	499	544	590	638	688	739	16
17	261	293	330	367	405	447	490	534	582	631	683	736	790	17
18	279	314	352	392	433	477	522	570	621	673	727	784	841	18
19	297	334	375	417	460	507	555	605	659	714	772	831	893	19
20	315	354	397	442	487	537	587	640	697	756	817	879	944	20
21	333	374	419	466	515	567	620	676	736	797	862	927	995	21
22	351	395	442	491	542	597	653	711	774	839	906	975	1045	22
23	369	415	464	516	570	627	685	746	813	880	951	1025	1095	23
24	387	435	487	541	597	657	718	782	851	922	996	1070	1150	24
25	407	457	512	568	627	689	753	820	892	966	1045	1120	1205	25
26	427	480	536	595	657	722	788	858	934	1010	1090	1175	1260	26
27	447	502	560	623	687	754	824	897	975	1060	1140	1225	1315	27
28	468	525	584	650	717	787	859	935	1020	1110	1185	1275	1365	28
29	488	547	608	678	747	819	894	974	1060	1160	1235	1330	1420	29
30	508	570	632	705	777	852	930	1010	1100	1205	1285	1380	1475	30
31	528	592	656	733	807	885	965	1050	1140	1255	1330	1430	1530	31
32	550	615	685	759	835	916	1000	1090	1180	1280	1375	1480	1585	32

TABLE I—Continued.

Length in feet	30	31	32	33	34	35	36	37	38	39	40	41	42	Length feet
	BOARD FEET													
8	383	410	438	469	498	528	561	594	627	664	698	734	774	8
9	434	465	497	531	564	598	635	672	710	752	790	831	876	9
10	486	520	555	594	630	668	710	751	793	839	882	929	1130	10
11	537	574	614	656	696	739	783	829	876	926	973	1024	1080	11
12	588	629	673	718	762	808	858	908	959	1015	1065	1120	1180	12
13	639	684	721	781	828	879	932	986	1040	1100	1155	1215	1280	13
14	690	739	790	843	894	949	1005	1065	1125	1185	1250	1315	1380	14
15	742	792	848	905	960	1020	1080	1140	1210	1275	1340	1410	1485	15
16	793	848	907	968	1025	1090	1155	1220	1290	1360	1430	1505	1585	16
17	848	907	969	1035	1095	1165	1235	1305	1375	1455	1530	1610	1690	17
18	903	965	1030	1100	1165	1235	1310	1385	1465	1545	1625	1710	1800	18
19	957	1025	1095	1165	1235	1310	1390	1470	1550	1635	1720	1810	1905	19
20	1010	1085	1155	1230	1305	1385	1470	1550	1640	1730	1820	1915	2010	20
21	1065	1140	1220	1300	1375	1460	1545	1635	1725	1820	1915	2015	2120	21
22	1120	1200	1280	1365	1450	1535	1625	1720	1810	1910	2010	2115	2225	22
23	1175	1260	1345	1430	1520	1610	1705	1800	1900	2005	2110	2220	2330	23
24	1230	1320	1405	1495	1590	1685	1780	1885	1985	2095	2205	2320	2435	24
25	1290	1380	1470	1565	1660	1760	1865	1970	2080	2190	2305	2425	2550	25
26	1350	1440	1535	1635	1735	1840	1950	2060	2170	2285	2410	2530	2660	26
27	1405	1505	1605	1705	1810	1920	2030	2145	2260	2385	2510	2640	2770	27
28	1465	1565	1670	1776	1885	1995	2115	2230	2355	2480	2610	2745	2880	28
29	1525	1630	1735	1845	1960	2075	2195	2320	2445	2580	2715	2850	2995	29
30	1585	1690	1800	1915	2030	2155	2280	2405	2535	2675	2815	2960	3105	30
31	1640	1750	1865	1990	2105	2230	2365	2495	2630	2770	2915	3065	3215	31
32	1700	1815	1935	2055	2180	2310	2445	2580	2720	2870	3015	3170	3325	32

TABLE I—Continued.

Length in feet	43	44	45	46	47	48	49	50	51	52	53	54	55	Length in feet
	DIAMETER IN INCHES — BOARD FEET													
8	812	852	890	932	976	1020	1065	1110	1155	1200	1250	1300	1345	8
9	919	963	1005	1055	1105	1150	1200	1255	1305	1355	1410	1465	1520	9
10	1025	1075	1125	1175	1230	1285	1340	1395	1455	1510	1575	1635	1695	10
11	1130	1185	1240	1300	1360	1415	1480	1540	1605	1670	1735	1805	1870	11
12	1240	1295	1355	1420	1485	1550	1615	1685	1755	1825	1900	1970	2045	12
13	1345	1410	1475	1540	1625	1685	1755	1830	1905	1980	2060	2140	2170	13
14	1450	1520	1590	1665	1740	1815	1895	1975	2055	2140	2225	2310	2395	14
15	1560	1630	1705	1785	1865	1950	2035	2120	2205	2290	2385	2475	2570	15
16	1660	1740	1820	1905	1990	2080	2170	2260	2355	2445	2545	2645	2745	16
17	1775	1860	1945	2035	2000	2220	2315	2410	2515	2610	2715	2820	2925	17
18	1885	1975	2065	2160	2260	2360	2460	2560	2670	2770	2880	2995	3105	18
19	1995	2090	2190	2290	2390	2495	2605	2710	2825	2935	3050	3170	3285	19
20	2105	2210	2310	2420	2525	2635	2750	2860	2980	3095	3220	3345	3470	20
21	2220	2325	2430	2545	2655	2775	2895	3010	3135	3260	3380	3520	3650	21
22	2330	2440	2555	2670	2790	2910	3040	3160	3295	3420	3565	3695	3730	22
23	2440	2555	2675	2795	2920	3050	3180	3310	3450	3585	3725	3870	4010	23
24	2550	2675	2800	2925	3055	3190	3325	3460	3605	3745	3890	4045	4195	24
25	2670	2795	2925	3060	3195	3335	3475	3620	3770	3915	4065	4225	4380	25
26	2785	2920	3050	3190	3330	3480	3625	3775	3930	4080	4240	4405	4570	26
27	2900	3040	3180	3325	3470	3625	3775	3930	4090	4250	4415	4585	4755	27
28	3020	3160	3305	3460	3610	3765	3925	4085	4255	4415	4590	4765	4945	28
29	3135	3285	3435	3590	3745	3910	4075	4240	4415	4585	4765	4950	5135	29
30	3250	3405	3560	3725	3885	4055	4225	4400	4580	4755	4940	5130	5320	30
31	3365	3530	3690	3855	4025	4200	4375	4555	4740	4920	5115	5310	5510	31
32	3485	3650	3815	3990	4165	4345	4530	4710	4905	5090	5290	5495	5695	32

TABLE I—Continued.

Length in feet	DIAMETER IN INCHES 56	57	58	59	60	61	62	63	64	65	66	67	68	Length in feet
	BOARD FEET													
8	1400	1450	1505	1560	1615	1665	1725	1780	1840	1900	1960	2020	2080	8
9	1580	1640	1700	1760	1820	1880	1945	2010	2075	2145	2210	2280	2350	9
10	1760	1825	1895	1960	2030	2095	2170	2240	2315	2390	2465	2540	2610	10
11	1945	2015	2000	2165	2240	2315	2390	2470	2550	2635	2715	2800	2885	11
12	2125	2200	2285	2365	2445	2525	2615	2700	2790	2880	2970	3060	3150	12
13	2305	2390	2475	2570	2655	2745	2840	2930	3030	3125	3220	3320	3420	13
14	2485	2580	2675	2770	2865	2960	3060	3160	3265	3370	3475	3580	3690	14
15	2670	2765	2865	2975	3075	3175	3285	3390	3500	3615	3730	3840	3960	15
16	2850	2950	3060	3170	3280	3390	3500	3620	3735	3855	3975	4100	4220	16
17	3035	3145	3265	3380	3495	3615	3730	3860	3980	4110	4235	4370	4500	17
18	3225	3340	3465	3585	3710	3835	3960	4095	4225	4360	4495	4640	4775	18
19	3415	3535	3665	3795	3930	4060	4190	4335	4470	4615	4755	4905	5050	19
20	3600	3730	3870	4005	4145	4280	4420	4570	4715	4865	5015	5175	5325	20
21	3790	3925	4070	4210	4360	4505	4650	4810	4960	5120	5275	5445	5600	21
22	3975	4120	4275	4420	4575	4730	4880	5045	5205	5370	5535	5710	5880	22
23	4165	4315	4475	4630	4790	4950	5110	5280	5450	5625	5795	5980	6150	23
24	4355	4510	4675	4835	5005	5170	5340	5520	5695	5875	6055	6245	6430	24
25	4545	4710	4885	5050	5230	5400	5580	5765	5950	6135	6325	6520	6715	25
26	4740	4915	5090	5265	5450	5630	5815	6010	6200	6395	6590	6800	7000	26
27	4935	5115	5300	5480	5675	5860	6055	6255	6455	6655	6860	7075	7285	27
28	5130	5315	5510	5695	5895	6090	6290	6500	6705	6915	7130	7350	7570	28
29	5325	5520	5715	5915	6120	6320	6530	6745	6960	7175	7395	7625	7855	29
30	5520	5720	5915	6130	6345	6550	6765	6990	7210	7435	7665	7900	8140	30
31	5715	5925	6125	6345	6565	6780	7005	7235	7465	7695	7935	8180	8425	31
32	5910	6120	6345	6560	6785	7010	7240	7480	7715	7960	8200	8455	8705	32

TABLE I—Continued.

Length in feet	DIAMETER IN INCHES 69	70	71	72	73	74	75	76	77	78	79	80	81	Length in feet
	BOARD FEET													
8	2145	2210	2280	2340	2405	2475	2540	2610	2685	2755	2825	2900	2970	8
9	2420	2495	2515	2640	2705	2795	2865	2945	3030	3110	3185	3270	3355	9
10	2695	2780	2835	2940	3025	3115	3195	3285	3375	3460	3550	3645	3735	10
11	2975	3065	3155	3245	3335	3430	3520	3620	3720	3815	3910	4015	4115	11
12	3250	3350	3450	3545	3645	3745	3845	3955	4065	4165	4275	4385	4500	12
13	3525	3630	3740	3845	3955	4065	4170	4290	4410	4520	4635	4755	4880	13
14	3800	3915	4035	4145	4265	4385	4500	4620	4755	4875	5000	5130	5260	14
15	4075	4200	4325	4445	4570	4700	4825	4960	5100	5225	5360	5500	5640	15
16	4350	4480	4610	4745	4880	5020	5150	5290	5440	5580	5720	5870	6020	16
17	4635	4775	4915	5055	5200	5350	5490	5635	5795	5945	6090	6255	6415	17
18	4920	5065	5215	5365	5520	5675	5825	5980	6150	6305	6465	6635	6805	18
19	5205	5360	5515	5675	5835	6000	6160	6325	6500	6670	6835	7015	7195	19
20	5490	5650	5815	5985	6150	6325	6495	6670	6855	7030	7205	7400	7585	20
21	5775	5945	6115	6295	6470	6655	6830	7010	7205	7395	7575	7780	7975	21
22	6060	6235	6415	6605	6790	6980	7165	7355	7560	7755	7950	8160	8370	22
23	6345	6525	6715	6910	7105	7305	7500	7700	7910	8115	8320	8540	8755	23
24	6630	6820	7020	7225	7425	7635	7840	8045	8265	8480	8690	8925	9150	24
25	6925	7125	7330	7540	7750	7970	8180	8400	8625	8855	9075	9315	9550	25
26	7215	7425	7640	7860	8075	8305	8525	8755	8990	9225	9455	9705	9950	26
27	7510	7725	7950	8180	8400	8640	8870	9105	9350	9595	9835	10100	10350	27
28	7800	8025	8260	8495	8730	8975	9215	9460	9715	9970	10200	10500	10750	28
29	8095	8325	8570	8815	9055	9310	9560	9810	10100	10350	10600	10900	11150	29
30	8385	8630	8880	9130	9380	9645	9905	10150	10450	10700	11000	11250	11550	30
31	8680	8930	9190	9450	9710	9985	10250	10500	10800	11100	11350	11650	11950	31
32	8965	9230	9495	9770	10050	10300	10600	10900	11150	11454	11750	12050	12355	32

TABLE I—Continued.

Length in feet	82	83	84	85	86	87	88	89	90	91	92	93	94	Length in feet
	DIAMETER IN INCHES — BOARD FEET													
8	3050	3130	3205	3285	3365	3445	3525	3605	3690	3775	3855	3940	4030	8
9	3445	3530	3615	3705	3795	3885	3975	4070	4160	4255	4350	4445	4545	9
10	3835	3930	4025	6125	4225	4325	4425	4530	4630	4735	4845	4950	5060	10
11	4225	4330	4435	4545	4655	4765	4880	4985	5105	5220	5335	5455	5570	11
12	4615	4730	4850	4965	5085	5205	5325	5450	5575	5700	5830	5955	6090	12
13	5010	5135	5230	5390	5515	5650	5775	5905	6050	6185	6325	6460	6600	13
14	5400	5535	5670	5810	5950	6085	6230	6370	6520	6665	6815	6965	7120	14
15	5790	5935	6080	6230	6375	6530	6680	6835	6990	7150	7305	7470	7630	15
16	6180	6335	6490	6650	6810	6965	7130	7295	7460	7630	7800	7970	8145	16
17	6580	6745	6910	7080	7250	7420	7590	7765	7945	8125	8305	8485	8670	17
18	6980	7155	7330	7510	7690	7870	8055	8235	8425	8615	8810	9000	9200	18
19	7380	7565	7755	7940	8130	8320	8515	8710	8910	9110	9310	9515	9725	19
20	7785	7975	8170	8370	8570	8770	8975	9180	9390	9600	9815	10050	10250	20
21	8190	8385	8595	8805	9015	9220	9435	9650	9870	10100	10300	10550	10800	21
22	8590	8800	9015	9235	9455	9675	9900	10100	10350	10600	10850	11050	11300	22
23	8985	9210	9435	9665	9895	10120	10350	10600	10850	11100	11300	11550	11850	23
24	9385	9620	9855	10100	10350	10520	10800	11050	11300	11600	11850	12100	12350	24
25	9795	10050	10300	10550	10800	11050	11300	11550	11800	12100	12350	12600	12900	25
26	10200	10450	10700	10950	11250	11500	11750	12050	12300	12600	12850	13150	13400	26
27	10600	10900	11150	11400	11700	11950	12250	12500	12800	13100	13350	13650	13950	27
28	11050	11300	11600	11850	12150	12400	12700	13000	13300	13600	13900	14200	14500	28
29	11450	11700	12000	12300	12600	12900	13200	13500	13800	14100	14400	14700	15050	29
30	11850	12150	12450	12750	13050	13350	13650	13950	14300	14600	14900	15250	15550	30
31	12250	12550	12850	13200	13500	13800	14100	14450	14800	15100	15450	15750	16100	31
32	12650	13000	13300	13600	13950	14250	14600	14950	15300	15600	15950	16300	16650	32

TABLE I—Continued.

Length in feet	95	96	97	98	99	100	101	102	103	104	105	106	107	Length in feet
	DIAMETER IN INCHES / BOARD FEET													
8	4115	4205	4295	4395	4480	4570	4665	4755	4855	4050	5045	5145	5240	8
9	4610	4740	4840	4955	5050	5155	5260	5365	5470	5580	5690	5800	5910	9
10	5170	5280	5390	5515	5620	5740	5855	5970	6090	6210	6330	6455	6575	10
11	5690	5800	5910	6075	6190	6320	6445	6580	6705	6840	6975	7110	7245	11
12	6220	6350	6495	6635	6765	6900	7040	7185	7330	7470	7680	7765	7915	12
13	6745	6885	7035	7195	7335	7480	7640	7795	7945	8095	8260	8420	8580	13
14	7270	7425	7590	7755	7905	8070	8235	8400	8565	8730	8900	9075	9245	14
15	7800	7965	8140	8315	8480	8650	8830	9005	9185	9360	9545	9730	9910	15
16	8320	8500	8680	8865	9045	9235	9420	9610	9800	9995	10200	10400	10600	16
17	8860	9050	9240	9435	9630	9830	10050	10250	10450	10650	10850	11050	11250	17
18	9395	9600	9800	10000	10210	10400	10650	10850	11050	11300	11500	11700	11950	18
19	9930	10150	10350	10600	10800	11000	11250	11450	11700	11900	12150	12400	12650	19
20	10450	10700	10900	11150	11400	11600	11850	12100	12300	12550	12800	13050	13300	20
21	11000	11250	11500	11700	11950	12200	12450	12700	12950	13200	13450	13750	14000	21
22	11550	11800	12050	12300	12550	12800	13050	13300	13600	13850	14100	14400	14650	22
23	12100	12350	12600	12850	13150	13400	13650	13950	14200	14500	14800	15250	15350	23
24	12600	12900	13150	13450	13700	14000	14250	14550	14850	15150	15450	15750	16050	24
25	13150	13450	13750	14000	14300	14600	14900	15200	15500	15800	16100	16400	16700	25
26	13700	14000	14300	14600	14900	15200	15500	15800	16150	16450	16750	17100	17400	26
27	14250	14550	14850	15200	15500	15800	16150	16450	16750	17100	17450	17750	18100	27
28	14800	15150	15450	15750	16100	16400	16750	17100	17400	17750	18100	18450	18800	28
29	15350	15700	16000	16350	16700	17050	17350	17700	18000	18400	18750	19150	19500	29
30	15900	16250	16600	16950	17300	17650	18000	18350	18700	19050	19450	19800	20200	30
31	16450	16800	17150	17500	17850	18250	18600	18950	19350	19700	20100	20500	20000	31
32	17000	17350	17750	18100	18450	18850	19200	19600	20000	20400	20750	21150	21550	32

TABLE I—Continued.

Length in feet	DIAMETER IN INCHES													Length in feet
	108	109	110	111	112	113	114	115	116	117	118	119	120	
	BOARD FEET.													
8	5345	5445	5545	5650	5750	5860	5965	6070	6175	6285	6395	6505	6615	8
9	6025	6140	6250	6370	6485	6605	6700	6840	6930	7080	7205	7330	7460	9
10	6705	6830	6955	7090	7215	7350	7500	7615	7745	7880	8025	8160	8300	10
11	7385	7520	7660	7810	7945	8095	8250	8385	8530	8675	8835	8985	9140	11
12	8065	8220	8370	8525	8680	8840	9000	9155	9320	9480	9645	9815	9980	12
13	8745	8910	9075	9250	9415	9585	9750	9930	10100	10250	10450	10645	10800	13
14	9425	9600	9780	9970	10150	10350	10500	10700	10900	11050	11250	11450	11650	14
15	10100	10300	10500	10700	10900	11100	11300	11500	11650	11850	12100	12300	12500	15
16	10800	11000	11200	11400	11600	11800	12050	12250	12450	12700	12900	13100	13350	16
17	11500	11700	11900	12150	12350	12600	12800	13050	13250	13500	13700	13950	14200	17
18	12150	12400	12650	12850	13100	13350	13550	13800	14050	14300	14550	14800	15050	18
19	12850	13100	13350	13600	13850	14000	14350	14600	14850	15100	15350	15650	15900	19
20	13550	13800	14050	14350	14600	14850	15100	15400	15650	15950	16200	16500	16750	20
21	14250	14500	14800	15050	15350	15600	15900	16150	16450	16750	17000	17300	17600	21
22	14950	15250	15500	15800	16100	16350	16650	16950	17250	17550	17850	18150	18450	22
23	15650	15900	16250	16550	16850	17100	17450	17750	18050	18350	18650	19000	19300	23
24	16350	16650	16950	17250	17550	17900	18200	18550	18850	19200	19500	19850	20200	24
25	17050	17350	17650	18000	18350	18650	19000	19350	19650	20000	20350	20700	21050	25
26	17750	18050	18400	18750	19100	19450	19800	20150	20500	20850	21200	21550	21950	26
27	18450	18800	19150	19600	19850	20200	20550	20900	21300	21650	22050	22400	22800	27
28	19150	19500	19850	20250	20600	21000	21350	21700	22100	22500	22850	23250	23650	28
29	19850	20250	20600	21000	21350	21750	22150	22500	22900	23300	23700	24100	24550	29
30	20550	20950	21350	21750	22100	22500	22900	23300	23750	24150	24550	25000	25400	30
31	21250	21650	22050	22500	22900	23300	23700	24100	24550	24950	25400	25850	26250	31
32	22000	22400	22800	23200	23650	24050	24500	24900	25350	25800	26250	26700	27150	32

A COMPARISON OF THREE DIFFERENT TYPES OF LOG RULES.

There are three distinct types of log rules now in general use. They are as follows: (a) Rules with a waste allowance varying directly as the barked area of the log and the volume of the log after the barked area allowance is made. (b) Rules with a waste allowance varying directly as the total volume of the log alone. (c) Rules with a waste allowance varying directly as the total volume of the log plus a constant.

When $D =$ diameter at small end inside bark in inches.
When $L =$ length of log in feet.
When $a =$ constant (in inches).
When $\pi = 3.1416$.
When $c =$ constant with limits of 0 and 1.
When B. M. = volume in board feet of manufactured product,
the three types may be expressed by these formulæ:

$$\text{(a)} \quad (1-c)\frac{\pi(D-a)^2}{4\times 12}L = \text{B.M.}$$

$$\text{(b)} \quad (1-c)\frac{\pi D^2}{4\times 12}L = \text{B.M.}$$

$$\text{(c)} \quad \left[(1-c)\frac{\pi D^2}{4\times 12} - b\right]L = \text{B.M.}$$

NOTE: The above formulæ are special cases of

$$\left[(1-c)\frac{\pi(D-a)^2}{4\times 12} - b\right]L = \text{B.M.}$$

In formula (a) the constant b equals zero, and the constant a has a positive value. Therefore, the curve $(1-c)\frac{\pi D^2}{4\times 12}L$ has been moved in a horizontal direction a units to the right of the origin.

In (b), $a=0$, and $b=0$, or the curve maintains its normal position.

In (c), $a=0$, and b has a positive value, or the curve has been moved in a vertical direction b units.

None of the log rules analyzed had values for both a and b such that one of them could not be easily eliminated. The Universal Log Rule, for instance, reduces to the following formula:

$$\left[(1-.20)\frac{\pi(D-1.591)^2}{4\times 12} - .1325\right]L = \text{B.M.}$$

The constant $b=.1325$ is so small that its effect upon the log rule is negligible. $(1-.20)\frac{\pi(D-1.6)^2}{4\times 12}L = \text{B.M.}$ gives values for this rule within 2 board feet, and is the formula listed below.

The following is a comparison of log rules which may be expressed in the form:

$$(1 - c)\,\frac{\pi\,(D - a)^2}{4 \times 12}\,L = \text{B.M.}$$

NOTE: The constant c is the fractional part of the log becoming sawdust after an allowance of a inches from the diameter has been made for slabs. It can be expressed in per cent by multiplying by 100, or moving the decimal point two places to the right. $(1 - c)$ in like manner is the fractional part allowed for the manufactured product.

Champlain:

$$(1 - .20)\,\frac{\pi\,(D - .8)^2}{4 \times 12}\,L = \text{B.M.}$$

Boughman Rotary Saw: (Original values slightly erratic)

$$(1 - .19)\,\frac{\pi\,(D - .87)^2}{4 \times 12}\,L = \text{B.M.}$$

Boughman Band Saw: (Original values slightly erratic)

$$(1 - .10)\,\frac{\pi\,(D - 1)^2}{4 \times 12}\,L = \text{B.M.}$$

Wilson: (Original values slightly erratic)

$$(1 - .193)\,\frac{\pi\,(D - 1)^2}{4 \times 12}\,L = \text{B.M}$$

Carey: (Original values slightly erratic)

$$(1 - .193)\,\frac{\pi\,(D - 1)^2}{4 \times 12}\,L = \text{B.M.}$$

Baxter:

$$(1 - .338)\,\frac{\pi\,(D - 1)^2}{4 \times 12}\,L = \text{B.M.}$$

Click: (Original values slightly erratic)

$$(1 - .236)\,\frac{\pi\,(D - 1.25)^2}{4 \times 12}\,L = \text{B.M.}$$

British Columbia:

$$(1 - .273)\,\frac{\pi\,(D - 1.5)^2}{4 \times 12}\,L = \text{B.M.}$$

Universal:

$$(1 - .20)\,\frac{\pi\,(D - 1.6)^2}{4 \times 12}\,L = \text{B.M.}$$

International:

$$(1 - .16)\,\frac{\pi\,(D - 1.62)^2}{4 \times 12}\,L = \text{B.M.}$$

(Applied to 4′ sections with taper allowance of 1″ in 8′, and constructed for $\frac{1}{8}$″ saw-kerf.)

Preston:

$$(1 - .20)\,\frac{\pi\,(D - 1.75)^2}{4 \times 12}\,L = \text{B.M.} \quad \text{(Small logs)}$$

$$(1 - .20)\,\frac{\pi\,(D - 1.5)^2}{4 \times 12}\,L = \text{B.M.} \quad \text{(Large logs)}$$

Doyle:

$$(1 - .045)\,\frac{\pi\,(D - 4)^2}{4 \times 12}\,L = \text{B.M.}$$

McKenzie:

$$\left[1 - \frac{k\,(w + t + k)}{(w + k)\,(t + k)}\right]\frac{\pi\,(D - a)^2}{4 \times 12}\,L + C = \text{B.M.}$$

Where k = saw-kerf in inches.
Where t = average thickness of lumber sawed, in inches.
Where w = average width of lumber sawed, in inches.
Where a = constant.
Where C = constant included to compensate for excessive taper in small logs.

To be applied to 8′ sections with taper allowance of e″ in f′.

It will be observed that of the above rules the Doyle and the Baxter are the two extremes. The Doyle rule has an enormous slab allowance with extremely small allowance for sawdust, (4.5%); where the Baxter rule has a small slab allowance and a very large allowance for sawdust, (33.8%).

Log rules of this form are correct in principle, and can be adapted to conditions existing at different mills, and to the character of the timber in different localities. The sawdust allowance, however, should not be fixed, but should depend upon the width of saw-kerf and the average dimensions of the lumber. The slab allowance should also be flexible, and should be determined by the timber to be sawed. Allowances for taper, excessive taper in small logs, shrinkage, etc., can be applied when making up a table based upon

$$(1 - c)\,\frac{\pi\,(D - a)^2}{4 \times 12}\,L = \text{B.M.}$$

This type of log rule can be represented diagramatically by drawing concentric circles of diameters D and $(D - a)$ respectively. The difference between the two rings will represent slab allowance. Draw a sector of the small circle with angle equal to $c \times 360°$. This will represent the sawdust allowance.

The following is a comparison of log rules which may be expressed in the form:

$$(1 - c)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

Constantine:

$$(1 - 0)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

Saco River: (Original values slightly erratic)

$$(1 - .276)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

Derby: (Original values slightly erratic)

$$(1 - .279)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

Square of Three-quarters:

$$(1 - .283)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

Partridge: (Original values slighty erratic)

$$(1 - .312)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

Vermont:

$$(1 - .363)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$$

NOTE: This rule gives the solid contents in board feet of the largest square timber contained in a log D'' in diameter inside bark at small end, and when divided by 12, becomes the formula for the Inscribed Square Rule, which actually gives the cubic contents of the largest square timber that can be sawed from a log of known length and diameter.

Stillwell: (Original values erratic)

$$(1 - .368)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$$

Ake:

$$(1 - .376)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$$

Square of Two-Thirds:

$$(1 - .435)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$$

NOTE: This formula, when divided by 12, is supposed to give, but does not give, the number of cubic feet of square timber that can be sawed from a log D'' in diameter at middle point inside bark. After the division by 12 is made, it is called the Two-Thirds Rule.

Orange River:

$$(1 - .491)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$$

Cumberland River:

$$(1 - .548)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$$

It is obvious that the Constantine rule has no allowance for either slabs or sawdust, and that all log rules which can be expressed in this form have a total waste allowance which is directly proportional to the total volume of the log, (taper not taken into consideration). The two extremes are the Constantine and the Cumberland River. The former with no allowance for waste whatever and the latter with an allowance of 54.8%.

There can not exist for different sized logs a constant ratio between volume sawed out at mill and volume in board feet as shown by a log rule of the above form. The principle is incorrect.

$(1 - c)\dfrac{\pi D^2}{4 \times 12} L = \text{B.M.}$ can be represented diagramatically by drawing a circle diameter D and then a sector of that circle with angle at center equal to $c \times 360°$. The area of the sector will represent the total waste allowance and the remaining area the lumber product.

The following is a comparison of log rules which may be expressed in the form:

$$\left[(1-c)\frac{\pi D^2}{4 \times 12} - b\right] L = \text{B.M.}$$

Bangor: (Original values slightly erratic)

$$\left[(1-.258)\frac{\pi D^2}{4 \times 12} - .5\right] L = \text{B.M.}$$

Boynton: (Original values erratic)

$$\left[(1-.350)\frac{\pi D^2}{4 \times 12} - .67\right] L = \text{B.M.}$$

Parsons: (Original values erratic)

$$\left[(1-.246)\frac{\pi D^2}{4 \times 12} - 1\right] L = \text{B.M.}$$

Warner: (Original values erratic)

$$\left[(1-.466)\frac{\pi D^2}{4 \times 12} - 1\right] L = \text{B.M.}$$

Spaulding: (Original values slightly erratic)

$$\left[(1-.266)\frac{\pi D^2}{4 \times 12} - 2\right] L = \text{B.M.}$$

Hannah: (Original values very erratic)

$$\left[(1-.266)\frac{\pi D^2}{4 \times 12} - 2\right] L = \text{B.M.}$$

Applies approximately to logs from 12″ to 42″ in diameter. This rule is very poorly constructed.

Wilcox: (Original values erratic)

$$\left[(1-.340)\frac{\pi D^2}{4 \times 12} - 2\right] L = \text{B.M.}$$

Finch and Apgar: (Original values very erratic)

$$\left[(1-.280)\frac{\pi D^2}{4 \times 12} - 2.5\right] L = \text{B.M.}$$

Ropp:

$$\left[(1 - .236)\frac{\pi D^2}{4 \times 12} - 3\right] L = \text{B.M.}$$

Scribner: (Original values very erratic)

$$\left[(1 - .266)\frac{\pi D^2}{4 \times 12} - 3\right] L = \text{B.M.}$$

Applies approximately to logs from 14″ to 75″, inclusive, in diameter. This rule is very poorly constructed.

Favorite: (Original values erratic)

$$\left[(1 - .285)\frac{\pi D^2}{4 \times 12} - 3\right] L = \text{B.M.}$$

Maine: (Original values slightly erratic)

$$\left[(1 - .222)\frac{\pi D^2}{4 \times 12} - .67\right] L = \text{B.M.}$$

(For small logs, 6″ to 15″, inclusive.)

$$\left[(1 - .222)\frac{\pi D^2}{4 \times 12} - 2\right] L = \text{B.M.}$$

(For logs 16″ to 48″, inclusive.)

Herring: (Original values slightly erratic)

$$\left[(1 - .392)\frac{\pi D^2}{4 \times 12} - 1\right] L = \text{B.M.}$$

(Small logs up to 30″.)

$$\left[(1 - .313)\frac{\pi D^2}{4 \times 12} - 5.5\right] L = \text{B.M.}$$

(For logs from 30″ to 42″, inclusive.)

Dusenbury: (Original values slightly erratic)
Practically the same as the Herring Log Rule.

Rules of this form will usually give a large per cent of mill overrun for small logs, due to the presence of the constant *b*. Intermediate logs will run below, hold up the scale or overrun, all depending upon the value of c in the rule used and the width of the saw-kerf. The effect of the constant b becomes small for intermediate sized logs, and is practically negligible for large ones. Large logs will run higher in per cent of mill overrun than the intermediate, since the slab allowance in this type of log rule increases directly as the volume of the log plus a constant. The principle is incorrect.

$\left[(1-c)\frac{\pi D^2}{4 \times 12} - b\right] L =$ B.M. can be represented diagramatically

by drawing two concentric circles, the larger one with diameter D and the smaller one with diameter sufficient to allow for b board feet; then drawing a sector forming an angle of $c \times 360°$ at the center. The area of the sector and the small circle will represent the waste allowance for slabs, sawdust, etc., while the remaining area will be the lumber product.

MISCELLANEOUS LOG RULES.

The Chapin, Northwestern, White and Ballon log rules have no definite underlying principles.

The Drew and the Forty-five were found to be of the form

$$\left[1-(c-eD)\right]\frac{\pi D^2}{4\times 12}L=\text{B.M.}$$

Where $c=$ constant less than 1 and greater than 0.

Where $e=$ constant much smaller than c and greater than 0.

Their formulæ are as follows:

The Forty-five rule:

$$\left[1-(.496-.00763D)\right]\frac{\pi D^2}{4\times 12}L=\text{B.M.}$$

The Drew Rule:

$$\left[1-(.450-.003D)\right]\frac{\pi D^2}{4\times 12}L=\text{B.M.}$$

In these rules the allowance for total wastage when expressed in per cent of the total contents of the log, taper not considered, decreases uniformly as the diameter increases. When $eD=c$, there is no allowance for wastage whatever. The Forty-five Log Rule allows for no wastage in logs 65″ in diameter and shows more volume for logs over 65″ than they actually contain. The Drew rule also shows a uniformly decreasing per cent of wastage, and for logs 150″ in diameter the waste allowance becomes zero. The principle of these rules is absolutely incorrect.

LOG RULES BASED ON STANDARDS.

Any log rule, constructed to show volume in board feet of lumber contained in logs of various lengths and diameters, which is based upon definite principles, may be reduced to what is called a standard log rule. The only difference between the ordinary log rule and its unlimited number of standards is in the unit of measure. A log of any specified dimensions may be chosen as the unit of measure, and so long as the underlying principles of both the standard and the rule expressing values in board feet are the same, there will always exist a definite relation between them and the one may be expressed in terms of the other by multiplying by a constant.

When d = Diameter in inches of the standard log and
l = Length in feet of the standard log,

log rules of the form $(1-c)\dfrac{\pi(D-a)^2}{4\times 12}L = \text{B.M.}$

become $\dfrac{(D-a)^2 L}{(d-a)^2 l} = V$, in standards.

Log rules of the form $(1-c)\dfrac{\pi D^2}{4\times 12}L = \text{B.M.}$

become $\dfrac{D^2 L}{d^2 l} = V$, in standards.

Log rules of the form $\left[(1-c)\dfrac{\pi D^2}{4\times 12} - b\right]L = \text{B.M.}$

become $\dfrac{(D^2-s)\,L}{(d^2-s)\,l} = V$, in standards.

All standard log rules now in use are based upon $\dfrac{D^2 L}{d^2 l} = \text{Vol.}$, in standards. Therefore, any one of them may be reduced to the form $(1-c)\dfrac{\pi D^2}{4\times 12}L = \text{B.M.}$, since $\dfrac{D^2 L}{d^2 l}\times \text{Const.} = (l-c)\dfrac{\pi D^2}{4\times 12}L$. Furthermore, it is evident that all standard rules of the same form bear a constant relation the one to the other, and any number of units of a certain standard rule may be reduced to units of any other standard of the same form by multiplying by the proper constant. For example, the Nineteen Inch Standard Rule, $\left(\dfrac{D^2 L}{19^2\times 13} = V, \text{ in standards}\right)$, may be applied to a large number of logs of different sizes, and the aggregate scale of these logs then given in 19″ standards, may be reduced to Blodgett, cube standards, etc., or to any of the following log rules expressing results in board feet: Constantine, Saco River, Derby, Square of Three-quarters, Partridge, Vermont, Stillwell, Ake, Square of Two-thirds, Orange River, or Cumberland River, by multiplying the aggregate by the proper constant. The result in every case will be precisely the same as though the logs were scaled separately by each of the rules. If, however, it is desired to reduce the aggregate scale of these logs now expressed in standards or in board feet, as the case may be, to board feet as shown by the Doyle Log Rule, for instance, the problem is impossible. There is no way of making the reduction. The logs will have to be scaled in accordance with the principles of the Doyle Log Rule in order to get such results. If only a single log were in question instead of a number of different sizes, it would be very easy to make such a reduction, but since there is no common ratio existing between the Doyle Log Rule (also other rules of that form) and the Nineteen Inch Standard (and others of its form) for logs of all sizes, the reduction can not be applied to more than one log or set of logs of equal diameters.

It is folly to compare results obtained by two logs rules of different forms as applied to logs of various sizes. It is evident that a comparison of the formulæ of such rules would reveal a great deal more. Values shown by log rules of different forms are not comparable, since their underlying principles are different. Any comparison made of such values only lead to confusion and really do more harm than good.

The following will illustrate how the Nineteen Inch Standard Rule may be reduced to other standards and also to any log rule giving values in board feet which is of the same form:

Given: The Nineteen Inch Standard Rule $\frac{D^2 L}{19^2 \times 13} = V$, in 19″ standards, and given: The Blodgett rule $\frac{D^2}{16^2} L = V$, in Blodgett standards, to find the common reducing factor c:

$$\frac{D^2 L}{19^2 \times 13} \times c = \frac{D^2 L}{16^2}$$

$$\frac{c}{19^2 \times 13} = \frac{1}{16^2}$$

$$c = \frac{19^2 \times 13}{16^2} = 18.33$$

Therefore, if a log or any number of logs of different sizes have been scaled by the Nineteen Inch Standard Rule, the results may be expressed in Blodgett standards by multiplying by 18.33, which is the number of Blodgett standards contained in a Nineteen Inch standard. The ratio holds constant regardless of the size of the logs.

In like manner, the reducing factors for all other standard rules may be obtained.

Given: The Nineteen Inch Standard Rule $\frac{D^2 L}{19^2 \times 13} = V$, in standards, and the Vermont rule $(1 - .363)\frac{\pi D^2}{4 \times 12} L =$ B.M. in board feet.

To find how many board feet as shown by the Vermont rule are equivalent to a standard of the Nineteen Inch rule:

$$(1 - .363)\,\frac{\pi\,19^2}{48} \times 13 = 195.5$$

Therefore, 195.5 board feet as shown by the Vermont rule equals one standard of the Ninteen Inch Standard Rule. This relation holds for all sized logs. In like manner, reducing factors for the Constantine, Saco River, Derby, Square of Three-quarters, Partridge, Stillwell, Ake, Square of Two-thirds, Orange River and Cumberland River rules may be obtained. All rules of the above form have definite reducing factors which apply to all logs, regardless of size, and to any aggregate scale representing any number of logs.

Given: The Nineteen Inch Standard Rule $\frac{D^2\,L}{19^2 \times 13} = V$, in standards, to find a log rule equivalent to it when one standard = 200 board feet:

$$(1 - c)\,\frac{\pi\,19^2}{4 \times 12} \times 13 = 200$$

$$1 - c = \frac{200 \times 48}{\pi \times 19^2 \times 13} = .650$$

$$c = .350$$

Therefore: $(1 - .350)\,\frac{\pi\,D^2}{4 \times 12}\,L =$ B.M. is an equivalent rule for the Nineteen Inch Standard when a standard unit is equal to 200 board feet. In like manner equivalent rules for other standard rules may be obtained when the value of the unit is given in board feet.

For instance, the Blodgett rule allows 10 board feet for the equivalent of one standard, and the resulting rule which is equivalent to the Bodgett under these conditions is

$$(1 - .405)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

$(1 - .423)\,\frac{\pi\,D^2}{4 \times 12}\,L =$ B.M. is the equivalent for the cube rule when its standard unit = 12 board feet.

It must be borne in mind that log rules of the form

$$(1 - c)\,\frac{\pi\,D^2}{4 \times 12}\,L = \text{B.M.}$$

are very poor rules for measuring the number of board feet of lumber that can be sawed from logs of different sizes, and that the three distinct types of rules discussed under the heading "A Comparison of Three Different Types of Log Rules" can have no common reducing factor for logs of different sizes, since the underlying principles are not the same. In the case of the standard rule based upon $\frac{D^2 L}{d^2 l} = V$, V is directly proportional to the square of the diameter of the log and also directly proportional to its length, whereas a log rule based upon correct principles has the volume in board feet vary directly as the diameter minus a constant squared, and directly as the length, with a taper correction applied to at least 8′ sections.

Standard log rules based upon $\frac{D^2 L}{d^2 l} = V$ are, however, excellent rules where a measurement proportional to the total contents of the log is desired. Such measures are applicable to logs which are to be made into pulp or whenever the total contents of the log is to be used. These rules do not take taper into consideration. They can be reduced to cubic feet by multiplying by a constant.

THE TRANSFORMATION OF VOLUME TABLES BASED UPON A GIVEN LOG RULE TO VOLUME TABLES BASED UPON OTHER RULES.

Volume tables constructed to show the number of board feet contained in trees of different merchantable lengths and diameters breasthigh, and based upon a log rule of the form

$$(1-c)\frac{\pi\,(D-a)^2}{4\times 12}L = \text{B.M.}$$

can be transformed to tables based upon other rules of the same form where the value of the constant *a* is the same. If the value of *a* is different in the rule to which the values are to be reduced, there is no way of accomplishing the transformation. For example, tables based upon the Baxter rule can be transformed to tables based upon the Boughman Band Saw rule by dividing each value in the former table by (1 — .338) and multiplying by (1 — .10). But tables based upon the Baxter rule cannot be transformed to ones based upon the Doyle rule, or on any other rule of that form where *a* is not the same as in the Baxter rule, or to forms where *a* does not enter, unless the average diameter of all portions of the bole is known thus making it possible to find the value of *D* for all logs in the tree.

Volume tables based upon rules of the form $(1-c)\frac{\pi\,D^2}{4\times 12}L = \text{B.M.}$ and also upon the form $\left[(1-c)\frac{\pi\,D^2}{4\times 12} - b\right]L = \text{B.M.}$ can be easily transformed from the one to the other. For example, a volume table based upon the Spaulding Log Rule, showing the average volume in board feet of trees of different diameters breasthigh and merchantable lengths can be transformed to a table based upon the Ropp rule by adding twice the average merchantable length shown in the table to each average value, and then dividing by (1 — .266) and multiplying by (1 — .236) and subtracting from each value thus obtained three times the merchantable length. The resulting table will then be based upon the Ropp rule, and the values therein will be the same as though the Ropp rule had been used for scaling the individual logs instead of the Spaulding rule. In like manner, any volume table based upon a log rule of the form $\left[(1-c)\frac{\pi\,D^2}{4\times 12} - b\right]L = \text{B.M.}$, can be transformed to a volume table based upon any other log rule of that form.

Again, a volume table based upon a log rule of the above form can be transformed to a volume table based upon any log rule of the form $(1-c)\frac{\pi\,D^2}{4\times 12}L = \text{B.M.}$ by adding to each value in the table $b \times$ the merchantable length, and then dividing by $(1-c)$ of the log rule upon which it is based and multiplying by the value of $(1-c)$ of the log rule to which the transformation is to be made. For example: A volume table based upon the Spaulding Log Rule showing average volume in board feet of trees of different diameters breasthigh and merchantable lengths can be transformed to a table based upon the Vermont rule by adding twice the average merchantable length to each of the values shown in the table, and then dividing the values thus obtained by (1 — .266) and multiplying by (1 — .363). The resulting table will then be based upon the Vermont rule. Should it be desirable to further transform the table to values in cubic feet of the Inscribed Square rule, divide all values by 12. This last reduction will show the volume in cubic feet of the square timbers that can be sawed from trees of different merchantable lengths and diameters breasthigh.

The total number of cubic feet inside bark contained in logs of trees measured for the original volume table based on the Spaulding Log Rule can be obtained by adding twice the average merchantable length to each value in the table and then dividing by (1 — .266) and dividing by 12. This reduction gives the volume in cubic feet of the total logs in each tree, without the taper of the various logs originally measured being taken into consideration.

To recapitulate: All volume tables based upon

$$(1-c)\frac{\pi (D-a)^2}{4 \times 12} L = \text{B.M.}$$

can be reduced to any other table based upon the same form of log rule where the constant a is the same as in the rule originally used in compiling the table.

All volume tables based upon rules of the form

$$(1-c)\frac{\pi D^2}{4 \times 12} L = \text{B.M., or } \left[(1-c)\frac{\pi D^2}{4 \times 12} - b\right] L = \text{B.M.}$$

can be reduced or transformed to volume tables based upon any log rule of either of these forms, and in all cases the resulting tables will be the same as though the individual rules had been applied to the original data.

Any volume table based upon one of the following rules can be transformed to a volume table based upon any of the other rules here given: Constantine, Saco River, Derby, Square of Three-fourths, Partridge, Vermont, Inscribed Square (which is the Vermont rule divided by 12), Sillwell, Ake, Square of Two-thirds, Two-thirds rule (which is the Square of Two-thirds Rule divided by twelve), Orange River, Cumberland River, Bangor, Boynton, Parsons, Warner, Spaulding, Wilcox, Ropp, Favorite, Nineteen Inch Standard, New Hampshire (or Blodgett), the Cube Rule, Twenty-two Inch Standard, Twenty-four Inch Standard, Seventeen Inch Rule.

NOTE: The Hannah, Finch and Apgar, and Scribner rules have been omitted in the above list since their original values appear too erratic to be included. The Maine, Herring and Dusenbury also have been omitted, since each of these rules have separate formulæ for small and large logs.

In like manner any volume table based upon

$$(1-c)\frac{\pi (D-a)^2}{4 \times 12} L = \text{B.M.}$$

can be transformed to other volume tables of the same form, provided the constant a is the same in rules under consideration.

The following tables illustrate how the transformations described above may be made:

TABLE 2. Average volume in board feet, as shown by the Spaulding Log Rule, contained in merchantable portion of immature western yellow pine trees of different merchantable lengths and diameters breast high.

TABLE 2.

Diameter breast-high in inches	Merchantable length (feet) 70	80	90	100	110	120	130	Diameter inside bark top log, inches	Height of stump, feet	Basis, number of trees
	Volume, based on the Spaulding Rule (bd. ft.)									
20	300	380	465	550	--------	--------	--------	6.6	1.2	11
21	325	405	495	580	--------	--------	--------	6.7	1.2	
22	350	435	530	630	730	--------	--------	6.7	1.2	39
23	380	475	570	680	780	--------	--------	6.8	1.2	
24	415	510	620	730	840	--------	--------	6.9	1.3	67
25	450	560	670	785	905	--------	--------	7.0	1.3	
26	490	605	725	845	975	1100	--------	7.1	1.3	92
27	--------	655	780	915	1050	1180	--------	7.1	1.3	
28	--------	710	845	980	1130	1270	1415	7.2	1.3	100
29	--------	--------	910	1060	1210	1365	1520	7.3	1.3	
30	--------	--------	980	1140	1300	1460	1630	7.4	1.3	65
31	--------	--------	--------	1225	1395	1565	1750	7.5	1.3	
32	--------	--------	--------	1310	1490	1675	1870	7.6	1.4	57
33	--------	--------	--------	1400	1585	1780	1990	7.7	1.4	
34	--------	--------	--------	1495	1695	1900	2125	7.8	1.3	29
35	--------	--------	--------	--------	1800	2020	2255	7.9	1.3	
36	--------	--------	--------	--------	1910	2140	2400	8.0	1.4	27
37	--------	--------	--------	--------	--------	2265	2550	8.2	1.4	
38	--------	--------	--------	--------	--------	2395	2700	8.5	1.5	7
39	--------	--------	--------	--------	--------	2525	2850	9.0	1.5	
40	--------	--------	--------	--------	--------	2660	3005	9.6	1.5	8
Total number of trees										502

This table is based upon the original measurements of 502 trees.

TABLE 3. (A transformation of Table 2.) Average volume in board feet, as shown by the Ropp Log Rule, contained in merchantable portion of immature western yellow pine trees of different merchantable lengths and diameters breasthigh.

TABLE 3.

Diameter breast-high in inches	Merchantable length (feet) — Volume, based on the Ropp Log Rule (bd. ft.): 70	80	90	100	110	120	130	Diameter inside bark top log, inches	Height of stump, feet	Basis, number of trees
20	248	322	402	481	--------	--------	--------	6.6	1.2	11
21	274	348	433	512	--------	--------	--------	6.7	1.2	
22	300	380	469	564	659	--------	--------	6.7	1.2	39
23	331	421	511	616	710	--------	--------	6.8	1.2	
24	368	458	563	668	772	--------	--------	6.9	1.3	67
25	404	509	615	725	841	--------	--------	7.0	1.3	
26	446	556	672	787	913	1036	--------	7.1	1.3	92
27	--------	608	730	861	992	1118	--------	7.1	1.3	
28	--------	666	796	929	1076	1211	1353	7.2	1.3	100
29	--------	--------	874	1011	1159	1310	1463	7.3	1.3	
30	--------	--------	938	1045	1252	1410	1578	7.4	1.3	65
31	--------	--------	--------	1182	1350	1520	1702	7.5	1.3	
32	--------	--------	--------	1271	1450	1632	1829	7.6	1.4	57
33	--------	--------	--------	1355	1550	1745	1952	7.7	1.4	
34	--------	--------	--------	1464	1663	1868	2094	7.8	1.3	29
35	--------	--------	--------	--------	1771	1995	2230	7.9	1.3	
36	--------	--------	--------	--------	1890	2120	2378	8.0	1.4	27
37	--------	--------	--------	--------	--------	2248	2535	8.2	1.4	
38	--------	--------	--------	--------	--------	2380	2690	8.5	1.5	7
39	--------	--------	--------	--------	--------	2520	2850	9.0	1.5	
40	--------	--------	--------	--------	--------	2630	3010	9.6	1.5	8
Total number of trees										502

This table was obtained by transforming the values in Table 2, based on the Spaulding Log Rule, to values shown here based upon the Ropp rule. The transformation was made in accordance with the underlying principles of both rules, and was accomplished as follows: To each value shown in Table 2 twice the merchantable length indicated at top of table was added. The new values thus obtained were divided by (1 — .266) and multiplied by (1 — .236), and three times the merchantable length subtracted. The resulting table is based upon the Ropp rule, and does not include any logs under 10″ in diameter, since logs below this size have been automatically discarded by the Ropp rule formula, which gives small negative results for logs under 8″ and small positive results for logs between 8″ and 10″. The negatives below 8″ and the positives between an 8″ and 10″ will about neutralize, thus giving a table which does not include logs below 10″ in diameter.

TABLE 4. (A transformation of Table 2.) Average values in board feet, as shown by the Vermont Log Rule, contained in merchantable portion of immature western yellow pine trees of different merchantable lengths and diameters breasthigh.

TABLE 4.

Diameter breast-high in inches	Merchantable length (feet)							Diameter inside bark top log, inches	Height of stump, feet	Basis, number of trees
	70	80	90	100	110	120	130			
	Volume, based on the Vermont Rule (bd. ft.)									
20	382	469	560	651	--------	--------	--------	6.6	1.2	11
21	404	491	587	677	--------	--------	--------	6.7	1.2	
22	426	517	617	721	825	--------	--------	6.7	1.2	39
23	452	552	652	764	868	--------	--------	6.8	1.2	
24	482	582	695	807	922	--------	--------	6.9	1.8	67
25	512	625	738	855	977	--------	--------	7.0	1.8	
26	547	665	786	908	1039	1164	--------	7.1	1.8	92
27	--------	708	834	969	1103	1232	--------	7.1	1.8	
28	--------	755	890	1025	1172	1311	1454	7.2	1.8	100
29	--------	--------	954	1094	1242	1395	1546	7.3	1.8	
30	--------	--------	1008	1163	1320	1478	1641	7.4	1.3	65
31	--------	--------	--------	1238	1403	1568	1746	7.5	1.3	
32	--------	--------	--------	1312	1486	1663	1850	7.6	1.4	57
33	--------	--------	--------	1390	1568	1754	1954	7.7	1.4	
34	--------	--------	--------	1471	1663	1860	2072	7.8	1.3	29
35	--------	--------	--------	--------	1754	1962	2182	7.9	1.3	
36	--------	--------	--------	--------	1850	2068	2310	8.0	1.4	27
37	--------	--------	--------	--------	--------	2175	2440	8.2	1.4	
38	--------	--------	--------	--------	--------	2287	2572	8.5	1.5	7
39	--------	--------	--------	--------	--------	2400	2705	9.0	1.5	
40	--------	--------	--------	--------	--------	2520	2835	9.6	1.5	8
Total number of trees										502

This table was obtained by transforming the values in Table 2, based upon the Spaulding Log Rule, to values shown here based upon the Vermont Rule. The transformation was made in the following manner: To each value shown in Table 2, twice the merchantable length indicated at top of table was added to each of the values. Each of the new values thus obtained was divided by (1 — .266) and multiplied by (1 — .363). The resulting values form the above table, and include all logs contained in the merchantable lengths. This table is the same as would have been obtained had the results been based directly upon the woods measurements.

Table 5. (A transformation of Table 2.) Average values in cubic feet as shown by the Inscribed Square Log Rule contained in the largest square timbers that can be sawed from the merchantable portion of immature western yellow pine trees of different merchantable lengths and diameters breasthigh.

TABLE 5.

Diameter breast-high in inches	Merchantable length (feet)							Diameter inside bark top log, inches	Height of stump, feet	Basis, number of trees
	70	80	90	100	110	120	130			
	Volume, based on the Inscribed Square Rule (cu. ft.)									
20	31.8	39.1	46.7	54.3				6.6	1.2	11
21	33.7	40.9	48.8	56.4				6.7	1.2	
22	35.5	43.1	51.4	60.0	68.7			6.7	1.2	39
23	37.6	46.0	54.3	63.6	72.3			6.8	1.2	
24	40.2	48.5	57.9	67.4	76.8			6.9	1.3	67
25	42.7	52.1	61.5	71.3	81.4			7.0	1.3	
26	45.6	55.4	65.5	75.7	86.5	97.0		7.1	1.3	92
27		59.0	69.5	80.7	92.0	102.7		7.1	1.3	
28		62.9	74.2	85.5	97.7	109.3	121.1	7.2	1.3	100
29			79.4	91.2	103.6	116.2	128.8	7.3	1.3	
30			84.0	97.0	110.0	123.0	136.8	7.4	1.3	65
31				103.2	117.0	130.7	145.4	7.5	1.3	
32				109.3	123.8	138.7	154.0	7.6	1.4	57
33				115.8	130.7	146.1	162.8	7.7	1.4	
34				122.6	138.7	155.0	172.5	7.8	1.3	29
35					146.2	163.5	182.0	7.9	1.3	
36					154.2	172.3	192.5	8.0	1.4	27
37						181.2	203.2	8.2	1.4	
38						190.8	214.0	8.5	1.5	7
39						200.0	225.3	9.0	1.5	
40						210.0	236.0	9.6	1.5	8
Total number of trees										502

Values in this table are indirectly based upon the measurements necessary for a compilation of Table 2. They were obtained by dividing values shown in Table 4 by the constant 12.

THE TRANSFORMATION OF THE SCALE OF A NUMBER OF LOGS IN THE AGGREGATE, BASED UPON A GIVEN LOG RULE, TO THE SCALE OF THE SAME LOGS IN THE AGGREGATE, BASED UPON ANOTHER LOG RULE.

The total volume of a number of logs of various sizes as shown by a log rule of the form $(1-c)\frac{\pi (D-a)^2}{4 \times 12} L = \text{B.M.}$ can be transformed to the volume as would be shown by another log rule of that form where the constant a is the same. For example: Should it be required to know the total volume in board feet of a trainload of logs of various sizes as would be shown by the Boughman Band Saw Rule when the aggregate scale based upon the Baxter Rule is known to be 320,000 board feet, the following steps are necessary: Divide 320,000 by $(1-c)$ of the Baxter rule, which is $(1-.338)$, and multiply by $(1-c)$ of the Boughman Band Saw Rule, which is $(1-.10)$. The result thus obtained which will be 435,000 is the same as would have been obtained had the Bowman rule been used for the original scale. Such transformations can not be made where the constant a in the two rules in question are not the same. Had the trainload of logs been scaled by a rule of the form $(1-c)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$ it would not be possible to make such a transformation, but it would be possible to transform the total scale to a new total based upon another rule of the same form. For example: If a trainload of logs should scale 300,000 board feet by the Square of Three-quarters rule, and it should be required to find the aggregate scale according to the Inscribed Square rule, the following procedure is all that is necessary: Divide 300,000 by $(1-.283)$ and multiply by $(1-.363)$ and then divide by 12. The final result, 32,000 cubic feet, is exactly the same as would have been obtained had the Inscribed Square rule been used for the original scale. In like manner, a transformation could have been made to a number of other rules of similar form.

Had the trainload of logs been originally scaled by a log rule of the form $\left[(1-c)\frac{D^2}{4 \times 12} - b\right] L = \text{B.M.}$, such as the Spaulding rule, a transformation to another rule of that form where b is the same could be accomplished by dividing by $(1-c)$ of the formula used and multiplying by $(1-c)$ of the formula to which the transformation is to be made. But, in cases where the value of the constant b is different in the log rules in question, no reduction can be made, unless the sum of the length of all the logs in the trainload be known. If the sum of all log lengths is known, it would then be possible to transform the total scale to other total scales based upon $\left[(1-c)\frac{\pi D^2}{4 \times 12} - b\right] L = \text{B.M.}$ or $(1-c)\frac{\pi D^2}{4 \times 12} L = \text{B.M.}$ whether the constant b is the same or different in the rules in question. Had the trainload of logs been originally scaled by the Spaulding Log Rule, or any other rule of similar form, where b has a value greater than 0, the transformation of the total scale to a total based on a log rule of the form $(1-c)\frac{\pi D^2}{4 \times 12}L = \text{B.M.}$ would be impossible unless the sum of the lengths of all logs in the trainload be known. Suppose, for example, the aggregate scale of a trainload of logs was 250,000 board feet by the Spaulding Log Rule, and the sum of all log lengths in the load was 12,000 linear feet, and it was required to know the total scale when based upon the Square of Two-thirds rule, the following operations are all that would be necessary: Add to 250,000 twice the sum of all log lengths, which would be 24,000, divide by (1 — .266) and multiply by (1 — .435). The resulting aggregate scale of the trainload of logs based on the Square of Two-thirds rule would then be 211,000 board feet, which is the same as would have been obtained had the Square of Two-thirds rule been originally applied.

SUMMARY.

No log rule will give an accurate measure of the lumber content of logs of various sizes that fails to properly combine all the factors encountered in converting logs into lumber. These factors are the same for all species under all milling conditions. The value of the factors alone increases or decreases according to the species and method of sawing, but the number of factors remain constant. As a result of failing to recognize the factors that must be combined in devising a properly constructed log rule, by failing to employ all of them, or by combining them improperly, there is no accurate log rule in use applicable to variable milling conditions. Any log rule capable of becoming a standard measure and susceptible of correction for certain variable factors must recognize a slab allowance proportional to the barked area of the log, and a sawdust allowance expressed as a definite per cent of the total volume of all logs, not including slabs. The per cent for sawdust is dependent upon the width of the saw-kerf and average dimensions of lumber to be sawed. Other factors to be taken into account are taper, shrinkage, normal crook and excessive taper in small logs, but these are of less importance than the two cited above.

The following log rules are constructed with a total wastage allowance proportional to the total volume of the log, regardless of size—taper not considered:

Constantine, Saco River, Derby, Square of Three-quarters, Partridge, Vermont, Stillwell, Ake, Square of Two-thirds, Orange River, Cumberland River. These rules are incorrect in principle, therefore no correction is possible.

Another group of rules is derived by substituting a waste allowance proportional to total volume, plus a constant for logs of different sizes—taper not considered. It would seem as though some effort had been made to correct the inaccuracy of the preceding group by adding a constant to compensate for waste occasioned by sawing logs of different sizes. The underlying principles of these rules are incorrect, however, and consequently their values cannot be properly adjusted. Such rules are the following:

Bangor, Boynton, Parsons, Warner, Spaulding, Hannah, Wilcox, Finch and Apgar, Ropp, Scribner, Favorite, Maine, Herring, Dusenbury.

Log rules with slab allowance varying directly as the barked area of logs of different sizes and with sawdust allowance directly as the volume after the slab allowance has been made are correct in principle, but are not necessarily correct measures. Rules of this type are as follows:

Champlain, Boughman's Rotary Saw, Boughman's Band Saw, Wilson, Carey, Baxter, Click, British Columbia, Universal, International, Preston, Doyle, McKenzie.

Of the preceding rules the Champlain, Universal, International and McKenzie are the only ones that are at all flexible to milling conditions and character of timber to be sawed. The Champlain and the Universal are the same, with the exception of the slab allowance, which in the case of the Universal is twice as great as for the Champlain. The sawdust allowance for both rules is made by allowing $\left(100 - \frac{100}{1+k}\right)$ per cent of the volume of the log (taper not included) for sawdust. This factor is correct for a gang saw with saws k'' thick and 1″ apart, but does not apply to any other milling conditions. Taper is not taken into consideration by either of these rules. Both rules have a fixed slab allowance, and the sawdust factor is affected by saw-kerf alone.

The International Log Rule also has a fixed slab allowance, and the sawdust allowance is unaffected by the dimensions of the lumber to be sawed. The value of this factor has been worked out for different gauge saws, and is the same regardless of dimensions of the manufactured product. The rule has a fixed taper allowance of ½″ in 4′, and tables compiled in accordance with the rule are based upon 4′ sections.

Since the analysis proved that no log rule now in use is universally applicable, a rule has been prepared and designated the McKenzie rule, which may be made to apply accurately to any set of conditions and at all times be susceptible to proper corrections made necessary by modifications of local methods employed.

This rule, with no allowance made for shrinkage and surfacing, is shown on page 19, and for convenience may be written:

$$\left[1-\frac{(w+k)\ (t+k)\ -wt}{(w+k)\ (t+k)}\right]\frac{\pi\,(D-a)^2}{4\times 12}\,L+C=\text{B.M.}$$

With an allowance for shrinkage and surfacing included, the rule complete becomes:

$$\left[1-\frac{(w+c+k)\ \ (t+b+k)-wt}{(w+c+k)\ \ (t+b+k)}\right]\frac{\pi\,(D-a)^2}{4\times 12}\,L+C=\text{B.M.}$$

Where b and c in inches, represent these allowances in thickness and width, respectively.

APPENDIX.

How to Adjust the McKenzie Log Rule to Conditions Existing at Any Mill.

This can best be shown by assuming a set of conditions and then reducing the rule from its general form to a special form in accordance with whatever the limitations imposed may be. For example, assume the following:

Mill output for period of three months:

150,000 bd. ft. of	1″ × 3″	cut	1 1/16″ × 3 1/8″
120,000 bd. ft. of	1″ × 4″	cut	1 1/16″ × 4 1/8″
180,000 bd. ft. of	1″ × 6″	cut	1 1/16″ × 6 1/8″
225,000 bd. ft. of	1″ × 8″	cut	1 1/16″ × 8 1/8″
700,000 bd. ft. of	1″ × 12″	cut	1 1/16″ × 12 1/4″
550,000 bd. ft. of	1″ × 14″	cut	1 1/16″ × 14 1/4″
300,000 bd. ft. of	1″ × 16″	cut	1 1/16″ × 16 1/4″
270,000 bd. ft. of	1″ × 18″	cut	1 1/16″ × 18 1/4″
180,000 bd. ft. of	6/4″ × 8″	cut	1 9/16″ × 8 1/8″
275,000 bd. ft. of	6/4″ × 10″	cut	1 9/16″ × 10 1/8″
500,000 bd. ft. of	6/4″ × 12″	cut	1 9/16″ × 12 1/4″
300,000 bd. ft. of	6/4″ × 14″	cut	1 9/16″ × 14 1/4″
275,000 bd. ft. of	6/4″ × 16″	cut	1 9/16″ × 16 1/4″
240,000 bd. ft. of	6/4″ × 18″	cut	1 9/16″ × 18 1/4″
600,000 bd. ft. of	2″ × 4″	cut	2 1/8″ × 4 1/8″
450,000 bd. ft. of	2″ × 6″	cut	2 1/8″ × 6 1/8″
225,000 bd. ft. of	2″ × 8″	cut	2 1/8″ × 8 1/8″
175,000 bd. ft. of	2″ × 10″	cut	2 1/8″ × 10 1/8″
200,000 bd. ft. of	2″ × 12″	cut	2 1/8″ × 12 1/4″
210,000 bd. ft. of	3″ × 3″	cut	3 1/8″ × 3 1/8″
270,000 bd. ft. of	3″ × 6″	cut	3 1/8″ × 6 1/8″
250,000 bd. ft. of	3″ × 12″	cut	3 1/8″ × 12 1/4″
300,000 bd. ft. of	4″ × 4″	cut	4 1/8″ × 4 1/8″
150,000 bd. ft. of	4″ × 6″	cut	4 1/8″ × 6 1/8″
375,000 bd. ft. of	5″ × 8″	cut	5 1/8″ × 8 1/8″
180,000 bd. ft. of	6″ × 6″	cut	6 3/16″ × 6 3/16″
120,000 bd. ft. of	6″ × 8″	cut	6 3/16″ × 8 3/16″
275,000 bd. ft. of	7″ × 9″	cut	7 3/16″ × 9 3/16″
250,000 bd. ft. of	8″ × 8″	cut	8 1/4″ × 8 1/4″
200,000 bd. ft. of	8″ × 12″	cut	8 1/4″ × 12 1/4″
375,000 bd. ft. of	8″ × 16″	cut	8 1/4″ × 16 1/4″
190,000 bd. ft. of	12″ × 12″	cut	12 1/4″ × 12 1/4″

Width of saw kerf = 1/8″

Average taper (not including butt logs or top logs)
= approx. 1/2″ in 8′

Average thickness of slabs and edgings at small end of logs = 5/8″

To determine a special form of

$$\left[1-\frac{(w+c+k)\,(t+b+k)-wt}{(w+c+k)\,(t+b+k)}\right]\frac{\pi\,(D-a)^2}{4\times 12}L+C=\text{B.M.}$$

which will conform to the above milling conditions and character of timber.

(*a*) The determination of the average value of

$$\left[1 - \frac{(w+c+k)\ (t+b+k) - wt}{(w+c+k)\ (t+b+k)}\right] \quad (A)$$

For 1″ × 3″ lumber cut 1 1/16″ × 3 1/8″

$w = 3,\quad c = 1/8 = .125,\quad k = 1/8 = .125$

$t = 1,\quad b = 1/16 = .0625$

$(w+c+k) = 3. + .125 + .125 = 3.25$

$(t+b+k) = 1 + .062 + .125 = 1.187$

$(w+c+k)\ (t+b+k) = 3.25 \times 1.187 = 3.86$

$wt = 1 \times 3 = 3$

$$\text{Then } (A) = 1 - \frac{3.86 - 3}{3.86} = 1 - .223 = .777$$

Therefore 150,000 bd. ft. represents 77.7% of the original material, or 22.3% has been forfeited to sawdust, shrinkage and surfacing in manufacturing 1″ × 3″ lumber, cut 1 1/16″ × 3 1/8″ when saw kerf = 1/8″

$$\frac{150{,}000}{.777} = 193{,}000$$ = the volume in bd. ft. of material actually used in producing 150,000 bd. ft. of 1″ × 3″ lumber (not including slabs and edgings).

With simliar determinations made for all other dimensions of lumber cut, we have:

150,000	bd. ft. of	1″ × 3″	cut	1 1/16″ × 3 1/8″	requiring	193,000	bd. ft. of solid material.
120,000	bd. ft. of	1″ × 4″	cut	1 1/16″ × 4 1/8″	requiring	151,000	bd. ft. of solid material.
180,000	bd. ft. of	1″ × 6″	cut	1 1/16″ × 6 1/8″	requiring	222,000	bd. ft. of solid material.
225,000	bd. ft. of	1″ × 8″	cut	1 1/16″ × 8 1/8″	requiring	276,000	bd. ft. of solid material.
700,000	bd. ft. of	1″ × 12″	cut	1 1/16″ × 12 1/4″	requiring	857,000	bd. ft. of solid material.
550,000	bd. ft. of	1″ × 14″	cut	1 1/16″ × 14 1/4″	requiring	672,000	bd. ft. of solid material.
300,000	bd. ft. of	1″ × 16″	cut	1 1/16″ × 16 1/4″	requiring	364,000	bd. ft. of solid material.
270,000	bd. ft. of	1″ × 18″	cut	1 1/16″ × 18 1/4″	requiring	327,000	bd. ft. of solid material.
180,000	bd. ft. of	6/4″ × 8″	cut	1 9/16″ × 8 1/8″	requiring	209,000	bd. ft. of solid material.
275,000	bd. ft. of	6/4″ × 10″	cut	1 9/16″ × 10 1/8″	requiring	317,000	bd. ft. of solid material.
500,000	bd. ft. of	6/4″ × 12″	cut	1 9/16″ × 12 1/4″	requiring	579,000	bd. ft. of solid material.
300,000	bd. ft. of	6/4″ × 14″	cut	1 9/16″ × 14 1/4″	requiring	346,000	bd. ft. of solid material.
275,000	bd. ft. of	6/4″ × 16″	cut	1 9/16″ × 16 1/4″	requiring	316,000	bd. ft. of solid material.
240,000	bd. ft. of	6/4″ × 18″	cut	1 9/16″ × 18 1/4″	requiring	275,000	bd. ft. of solid material.
600,000	bd. ft. of	2″ × 4″	cut	2 1/8″ × 4 1/8″	requiring	718,000	bd. ft. of solid material.
450,000	bd. ft. of	2″ × 6″	cut	2 1/8″ × 6 1/8″	requiring	529,000	bd. ft. of solid material.
225,000	bd. ft. of	2″ × 8″	cut	2 1/8″ × 8 1/8″	requiring	261,000	bd. ft. of solid material.
175,000	bd. ft. of	2″ × 10″	cut	2 1/8″ × 10 1/8″	requiring	202,000	bd. ft. of solid material.
200,000	bd. ft. of	2″ × 12″	cut	2 1/8″ × 12 1/4″	requiring	232,000	bd. ft. of solid material.
210,000	bd. ft. of	3″ × 3″	cut	3 1/8″ × 3 1/8″	requiring	246,000	bd. ft. of solid material.
270,000	bd. ft. of	3″ × 6″	cut	2 1/8″ × 6 1/8″	requiring	304,000	bd. ft. of solid material.
250,000	bd. ft. of	3″ × 12″	cut	3 1/8″ × 12 1/4″	requiring	279,000	bd. ft. of solid material.
300,000	bd. ft. of	4″ × 4″	cut	4 1/8″ × 4 1/8″	requiring	340,000	bd. ft. of solid material.
150,000	bd. ft. of	4″ × 6″	cut	4 1/8″ × 6 1/8″	requiring	166,000	bd. ft. of solid material.
375,000	bd. ft. of	5″ × 8″	cut	5 1/8″ × 8 1/8″	requiring	405,000	bd. ft. of solid material.
180,000	bd. ft. of	6″ × 6″	cut	6 3/16″ × 6 3/16″	requiring	199,000	bd. ft. of solid material.
120,000	bd. ft. of	6″ × 8″	cut	6 3/16″ × 8 3/16″	requiring	131,000	bd. ft. of solid material.
275,000	bd. ft. of	7″ × 9″	cut	7 3/16″ × 9 3/16″	requiring	297,000	bd. ft. of solid material.
250,000	bd. ft. of	8″ × 8″	cut	8 1/4″ × 8 1/4″	requiring	277,000	bd. ft. of solid material.
200,000	bd. ft. of	8″ × 12″	cut	8 1/4″ × 12 1/4″	requiring	216,000	bd. ft. of solid material.
375,000	bd. ft. of	8″ × 16″	cut	8 1/4″ × 16 1/4″	requiring	402,000	bd. ft. of solid material.
190,000	bd. ft. of	12″ × 12″	cut	12 1/4″ × 12 1/4″	requiring	202,000	bd. ft. of solid material.

9,000,000 bd. ft. is manufactured from..........................10,510,000 bd. ft. of solid material, not including the wastage necessary for slabs and edgings.

10,510,000 — 9,060,000 = 1,450,000 bd. ft. required for sawdust, shrinkage and surfacing.

$\frac{1,450,000}{10,510,000}$ = .138 = fractional part of the logs, after slab allowance has been made, which becomes waste.

(1 — .138) = fractional part becoming lumber.

Therefore the average value of (A) becomes (1 — .138) for the above milling conditions.

(*b*) The determination of slab allowance or surface wastage:

This allowance is provided for in the formula by the constant "a", which represents twice the average thickness of the slabs and edgings coming from the small end of logs, regardless of their length. The value of "a" can be closely estimated at any mill by watching the logs being sawed into lumber. If the character of the timber being sawed is such that a waste allowance, additional to that made for slabs and edgings is necessary, to correct for losses due to crook, such an allowance should be made by increasing the value of the factor "a" to a sufficient amount to offset losses caused by such defects.

For the milling conditions under consideration here, the value of "a" is assumed to be 5/8″ × 2, or 1.25. Substituting this value and the average value of (A), already determined, in the general formula, we have the following special form:

$$(1 - .223)\frac{\pi (D - 1.25)^2}{4 \times 12} L + C = \text{B.M.}$$

for logs L feet long with no allowance made for taper.

For 8′ sections this form becomes:

$$(1 - .223)\frac{\pi (D - 1.25)^2}{6} + C$$

or

$$.407\ (D - 1.25)^2 + C = \text{B.M.}$$

The constant C is included in the formula to counteract excessive taper in small logs, and its value should never be over 10 board feet. It can be definitely determined for a certain class of timber, by first ascertaining the mill overrun for small logs when $C = o$, and then making the value of C great enough to correct for the overrun. Large logs will be affected a negligible amount by the addition of this small quantity.

With $C = 3$ board feet, we have for the final reduction of the general rule:

$$.407\ (D - 1.25)^2 + 3 = \text{B.M.}$$

to be applied to 8′ sections with a taper of 1/2″ in each 8′.

A volume table based on the above rule with a taper allowance of 1/2″ in 8′ should be compiled as follows:

Length in feet	DIAMETER IN INCHES									
	6	7	8	9	10	11	12	13	14	15
	BOARD FEET									
8	12	16	22	28	34	42	50	59	69	80
9	14	18	25							
10	16	21	28							
11	17	23	31							
12	19	26	34							
13	21	28	38							
14	22	30	41							
15	24	33	44							
16	26	35	47	59	72	96	104	123	144	
17	28	38	50							
18	30	41	54							
19	32	43	57							
20	34	46	61							
21	36	49	64							
22	38	51	68							
23	40	54	71							
24	42	57	75	93	114	148	163	192	224	

Values for 8′ sections of different diameters are first determined directly from the formula. Then 16′ logs are considered as being made up of two 8′ sections, the one being one-half inch in diameter greater than the other; 24′ logs as three 8′ sections, one of them being the measured diameter at small end of log, another one, one-half inch greater than this, and the third, one inch greater. Thus, 26 board feet, which is the volume given in the above table for a log 16′ long and 6″ in diameter, was obtained by adding 12 board feet, which is the volume given for an 8′ section of same diameter, and 14 board feet obtained by averaging twelve and sixteen. (The average of 12 and 16 board feet gives volume for 8′ section, six and one-half inches in diameter.) The volume of the 24′ log of six inches in diameter shown in the table was obtained by adding 26 and 16. Twenty-six board feet being the volume of the first two 8′ sections contained in the log and sixteen board feet being the volume of the third or largest section. Other values may be obtained in a similar manner.

If the taper allowance were 1″ in 8′ instead of 1/2″ in 8′, a 16′ log 6″ in diameter at the small end would scale the same as two 8′ sections; the one 6″ in diameter and the other 7″. A 24′ log 20″ in diameter would, in like manner, scale the same as three 8′ sections; the first 20″, the second 21″ and the third 22″ in diameter. If this log were 22′ long instead of 24′ the scale would then be equal to that of the first two sections plus three-quarters of the third. By similar computations, all values composing a complete volume table for logs of different diameters and lengths can be compiled.

Log rules determined as explained in this Appendix apply to average conditions existing at the mills where they are made and are average rules which do not measure the fluctuations encountered in individual logs.

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